HEAT REMOVAL FROM NUCLEAR REACTORS

OBJECTIVES: DISCUSS THERMAL DESIGN OF NUCLEAR REACTORS

- FISSION: ORIGIN OF THERMAL ENERGY.
- HEAT TRANSFER MECHANISMS:
  - ► CONDUCTION THROUGH FUEL RODS,
  - ► CONVECTION TO THE COOLANT.

- THE COOLANT TRANSPORTS THE THERMAL ENERGY TO **BOILERS** WHERE STEAM IS GENERATED.
  - ► HIGH COOLANT TEMPERATURE FOR HIGH STEAM CYCLES EFFICIENCY.
  - ► CONSEQUENCE: HIGH FUEL AND CLADDING TEMPERATURES - METALLURGICAL LIMITATIONS.

- HIGH POWER GENERATION ⟶ HIGH TEMPERATURE IN THE FUEL & CLADDING ~~TEMPERATURE.~~
  - ► METALLURGICAL LIMITATIONS .

- COOLANT IS CIRCULATED BY A PUMP.
  - ► W_P / Q_R MINIMUM.

- PRACTICALLY, NO LIMIT TO THE THERMAL POWER RELEASED IN THE FUEL - REACTOR CORE.

  QUESTION: **HOW TO EXTRACT IT WHILE RESPECTING TEMPERATURE LIMITATIONS?**

- THIS CAN BE ACHIEVED BY:
  - ► INCREASING THE COOLANT FLOW RATE.
    PROBLEMS: HIGH PUMPING POWER, VIBRATIONS.
  - ► INCREASING THE FLOW SECTION.
    PROBLEM: NEUTRON MULTIPLICATION FACTOR.
  - ► BY USING DIFFERENT COOLANT.
    REPLACE GAS COOLANTS BY LIQUID COOLANTS - LIGHT OR HEAVY WATER, OR BY LIQUID METALS.
  - ► USING HIGH FUEL-COOLANT CONTACT AREA.
    ROD BUNDLE DESIGN.
  - ► USING BOILING PHENOMENON.
- A COMPROMISE BETWEEN NUCLEAR AND THERMAL DESIGN IS REQUIRED.

GENERATION OF HEAT IN NUCLEAR REACTORS

THERMAL ENERGY RELEASE DURING FISSION IS DUE:

- FISSION PROCESS - MAINLY.
- NON-FISSION CAPTURE IN THE FUEL, MODERATOR, COOLANT MATERIAL & STRUCTURAL MATERIAL.
- **EMITTED** ENERGY DURING FISSION: 207 MeV.
- DISTRIBUTION OF EMITTED ENERGY AMONG DIFFERENT FISSION COMPONENTS - TABLE 7.1.
- ENERGY CONVERTED INTO HEAT- **RECOVERABLE ENERGY** : 200 MeV - TABLE 7.1.
  - NEUTRINOS - NO ENERGY DEPOSIT.
  - CAPTURE γ-RAYS - AN AVERAGE OF 5 MeV ENERGY DEPOSIT.
- DISTRIBUTION OF HEAT RELEASE AMONG DIFFERENT REACTOR COMPONENTS - TABLE 7.2.

Table 7.1 Emitted and recoverable energies for fission of ^{235}U

Components	Emitted energy	Energy converted to heat or recovered energy	
	Mev	Mev	% of total
Fission fragments	168	168	84
Neutrons	5	5	2.5
Prompt γ rays	7	7	3.5
Delayed radiations:			
β–rays	8	8	4
γ-rays	7	7	3.5
neutrinos	12	-	-
Capture γ-rays	-	5	2.5
TOTAL	207	200	100

Table 7.2 Distribution of heat release among different components of CANDU reactors

Components	Released thermal energy, %
Energy released in the fuel	93.9
Energy released in the pressure tubes	0.3
Energy released in the calandria	0.1
Energy released in the coolant	0.5
Energy released in the moderator	5.0
Energy released in the shieldings	0.2
TOTAL	100.0

VOLUMETRIC HEAT GENERATION RATE IN REACTOR FUEL

$$q'''(\vec{r}) = \alpha\, E_R \int_0^\infty N_f \sigma_f(E) \phi(\vec{r}, E)\, d\, E$$

$$\Sigma_f(E) = N_f\, \sigma_f(E)$$

$$q'''(\vec{r}) = \alpha\, E_R \int_0^\infty \Sigma_f\,(E) \phi(\vec{r}, E)\, d\, E$$

$$\overline{\sigma}_f = \frac{\int_0^\infty \sigma_f(E)\, \phi(\vec{r}, E) dE}{\int_0^\infty \phi(\vec{r}, E) dE}$$

$$q'''(\vec{r}) = \alpha\, E_R N_f \overline{\sigma}_f \int_0^\infty \phi(\vec{r}, E)\, dE = \alpha\, E_R \overline{\Sigma}_f \int_0^\infty \phi(\vec{r}, E)\, dE$$

FOR THERMAL NEUTRONS

$$\phi(\vec{r}, E) \Rightarrow \phi(\vec{r})$$

$$q'''(\vec{r}) = \alpha\, E_R\, \overline{\Sigma}_f\, \phi(\vec{r}) \quad MeV\, /\, cm^3$$

- E_R : RECOVERABLE FISSION ENERGY, 200 MeV.
- α : FRACTION OF THE FISSION ENERGY DEPOSITED IN THE FUEL.
- $\overline{\Sigma}_f$: MACROSCOPIC FISSION CROSS-SECTION OF THERMAL NEUTRONS.
- $\phi(\vec{r})$: THERMAL NEUTRONS FLUX.
- TO EXPRESS q''' IN

$$W / cm^3 \quad multiply \;\; by \;\; 1.602 \bullet 10^{-13}$$

$$kW / m^3 \quad multiply \;\; by \;\; 1.602 \bullet 10^{-10}$$

USEFUL DEFINITIONS:

- POWER DENSITY OF THE CORE: $kW / l \; or \; kW / m^3$
- SPECIFIC POWER OF THE FUEL: $kW / kg \; or \; MW / ton$
- POWER DENSITY OF THE FUEL: $kW / l \; or \; MW / m^3$

FOR CANDU REACTORS:

- AVERAGE POWER DENSITY OF THE CORE: 12 MW / m^3
- AVERAGE POWER DENSITY OF THE FUEL: 115 MW / m^3
- MAXIMUM POWER DENSITY OF THE FUEL: 190 MW / m^3

TABLE ~~3.1~~ 7.3

Table 7.3 Power density of popular reactors in MW/m^3

Reactor type	Average power density of the core	Average power density of the fuel*	Maximum power density of the fuel*
CANDU	12	110	190
Boiling water reactors	56	56	180
Pressurized water reactors	95-105	95-105	190-210

* Includes the coolant within the rod assemblies.

SPATIAL VOLUMETRIC HEAT GENERATION DISTRIBUTION IN THE REACTOR CORE

- WE HAVE SEEN THAT:

$$q'''(\vec{r}) = \alpha E_R \overline{\Sigma}_f \, \phi(\vec{r}) \sim \phi(\vec{r})$$

- $\phi(\vec{r})$ IS DETERMINED BY SOLVING:
  - MULTI-GROUP NEUTRON DIFFUSION EQUATION, OR
  - NEUTRON TRANSPORT EQUATION.
    (WHICH ARE BEYOND THE SCOPE OF THIS COURSE.)

- TO SIMPLIFY THE DISCUSSION, THE FOLLOWING ASSUMPTIONS ARE MADE.
  - CYLINDRICAL REACTOR CORE WITH AXIAL AND RADIAL REFLECTORS.
  - CONSTANT FUEL ENRICHMENT.
  - UNIFORMLY DISTRIBUTED FUEL THROUGHOUT THE CORE, I.E., HOMOGENEOUS REACTOR.

DISCUSS THE SITUATION FOR **PWR** AND **CANDU** REACTORS.

- UNDER ABOVE CONDITIONS, THE DISTRIBUTION OF THE THERMAL NEUTRONS IN THE REACTOR CORE IS APPROXIMATE BY:

$$\phi(r,z) = \phi_o \, J_o\left(2.405\frac{r}{R'}\right) cos\left(\pi\frac{z}{H'}\right)$$

- DISCUSS FIGURE 7.1 TO SHOW THAT THE APPROXIMATION IS REASONABLE.

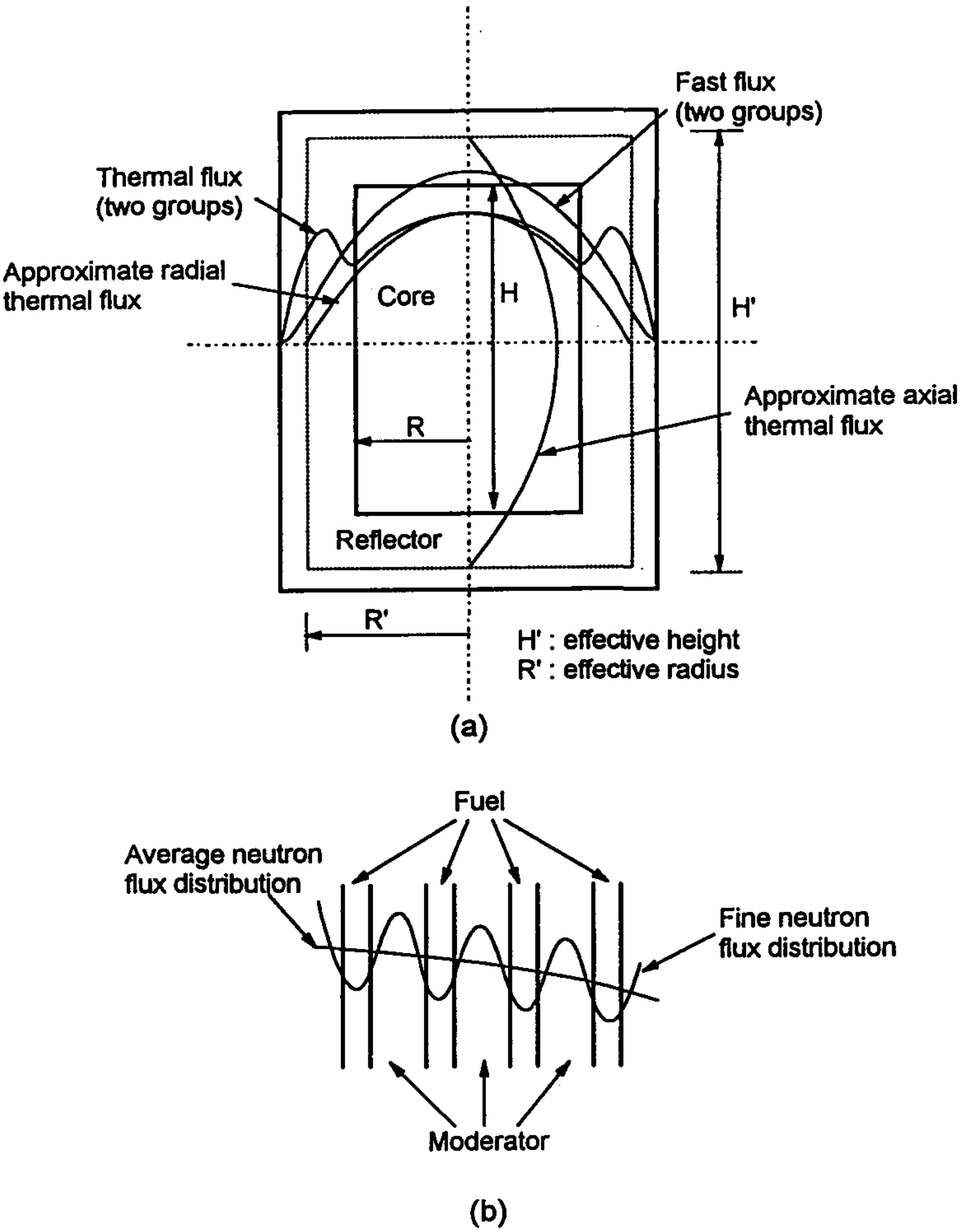

Figure 7.1 Comparison of radial thermal neutron distribution obtained with Eq. 7.7 and with two neutron energy groups

AVERAGE FLUX IN THE CORE REGION:

$$\bar{\phi} = \frac{1}{\pi R^2 H} \int_0^R \int_{-\frac{H}{2}}^{\frac{H}{2}} \phi \, 2\pi r \, dr \, dz$$

$$\phi(r,z) = \phi_o \, J_o\left(2.405 \frac{r}{R'}\right) cos\left(\pi \frac{z}{H'}\right)$$

INTEGRATION

$$\frac{\bar{\phi}}{\phi_o} = \frac{2RR'J_1\left(2.405\frac{R}{R'}\right)}{2.405R^2} \bullet \frac{2H' \, sin\left(\frac{\pi}{2}\frac{H}{H'}\right)}{\pi H}$$

SINCE $\phi(\vec{r}) \sim q'''(\vec{r})$

$$\frac{\bar{q}'''}{q_o'''} = \frac{2RR'J_1\left(2.405\frac{R}{R'}\right)}{2.405R^2} \bullet \frac{2H' \, sin\left(\frac{\pi}{2}\frac{H}{H'}\right)}{\pi H}$$

$\bar{q}'''$ AVERAGE HEAT GENERATION rate PER UNIT VOLUME OF THE CORE. power density of the core.

q_o''' MAXIMUM HEAT GENERATION rate WITHIN THE REACTOR.

$$\frac{\bar{q}'''}{q_o'''} = \frac{2RR'J_1\left(2.405\frac{R}{R'}\right)}{2.405R^2} \bullet \frac{2H'\sin\left(\frac{\pi}{2}\frac{H}{H'}\right)}{\pi H}$$

- FOR A BARE REACTOR WHERE $R = R'$ *and* $H = H'$ THE INVERSE OF THE ABOVE RATIO IS:

$$\frac{q_o'''}{\bar{q}'''} = \frac{2.405}{2J_1(2.405)}\frac{\pi}{2} = 3.64$$

- FOR

$$\frac{R}{R'} = \frac{H}{H'} = 0.83$$

THIS RATIO IS:

$$\frac{q_o'''}{\bar{q}'''} = 2.35$$

- FOR A UNIFORM RADIAL NEUTRON FLUX:

$$\frac{q_o'''}{\bar{q}'''} = 1.57$$

CONCLUSION: THE MAXIMUM HEAT GENERATION RATE IN THE CORE MAY BE SUBSTANTIALLY HIGHER THAN THE AVERAGE HEAT GENERATION RATE.

NEUTRON FLUX FLATTENING

- HIGH $q_o'''/\bar{q}'''$ IS NOT DESIRABLE.
- REACTOR POWER OUTPUT CAN BE INCREASED BY DECREASING THIS RATIO - I.E., MORE UNIFORM HEAT GENERATION RATE.
- THIS IS KNOWN AS: "FLATTENING OF THE HEAT GENERATION DISTRIBUTION OR ~~OF~~ THE NEUTRON FLUX DISTRIBUTION.
- NEUTRON FLUX FLATTENING IS ACHIEVED BY:
  - ADDING REFLECTORS AROUND A BARE REACTOR.
  - VARYING RADIAL FUEL ENRICHMENT.
  - VARYING THE RATIO OF FUEL-TO-MODERATOR.
  - VARYING THE EXTEND OF INSERTION OF CONTROL RODS.
  - USING A NEUTRON POISON IN CERTAIN PARTS OF THE CORE.
- ADVANTAGES OF NEUTRON FLATTENING:
  - REACTOR POWER OUTPUT INCREASES.
  - IN CANDU REACTORS THE COOLANT DISTRIBUTION AMONG THE FUEL CHANNELS IS EASILY ACHIEVED.

DETERMINATION OF THE MAXIMUM NEUTRON FLUX

- ϕ_O APPEARING IN

$$\phi(r,z) = \phi_o\, J_o\left(2.405\frac{r}{R'}\right)cos\left(\pi\frac{z}{H'}\right)$$

IS NOT KNOWN A PRIORI

- ϕ_O IS DETERMINED BY USING THE TOTAL POWER OF THE REACTOR $\mathbf{P}_t$
- $\mathbf{P}_t$ IS ALSO GIVEN BY

$$\boxed{\mathbf{P}_t = \int_{-\frac{1}{2}H}^{\frac{1}{2}H}\int_0^R q'''(r,z)\,2\pi r\, dr\, dz}$$

homogeneous
reactor.

$$\boxed{q'''(\vec{r}) = \alpha\; E_R\; \overline{\Sigma}_f\; \phi(\vec{r})}$$

$$\boxed{\phi(r,z) = \phi_o\, J_o\left(2.405\frac{r}{R'}\right)cos\left(\pi\frac{z}{H'}\right)}$$

$$\boxed{\overline{\Sigma}_f = \Sigma_{fm}}$$

Σ_{fm} : MACROSCOPIC FISSION CROSS-SECTION OF THE MIXTURE FOR THERMAL NEUTRONS.

$$\boxed{\mathbf{P}_t = \alpha\, E_R \overline{\Sigma}_{fm} \int_{-\frac{1}{2}H}^{\frac{1}{2}H}\int_0^R \phi_o\, J_o\left(2.405\frac{r}{R'}\right)cos\,\pi\frac{z}{H'}\,2\pi r\, dr\, dz}$$

$$\boxed{\alpha \cong 1}$$

$$\boxed{\int r\, J_o(\lambda r)\, dr = \frac{r}{\lambda} J_1(\lambda r)}$$

$$\boxed{\phi_o = \frac{\mathbf{P}_t}{E_R \overline{\Sigma}_{fm} \frac{R R'}{2.405} 4 H' J_1\left(2.405 \frac{R}{R'}\right) \sin \frac{\pi}{2} \frac{H}{H'}}}$$

WHERE

$\overline{\Sigma}_{fm}$	IS THE MACROSCOPIC FISSION CROSS-SECTION FOR THERMAL NEUTRONS
E_R	RECOVERED ENERGY

IF

$$\frac{R}{R'} = \frac{H}{H'} = 0.83$$

ϕ_o BECOMES:

$$\phi_o = 2.35 \frac{\mathbf{P}_t}{E_R \overline{\Sigma}_{fm} V}$$

THE DISTRIBUTION OF THE THERMAL NEUTRON FLUX IS THEN GIVEN BY:

$$\phi = \frac{2.35 \mathbf{P}_t}{E_R \overline{\Sigma}_{fm} V} J_o\left(2.405 \frac{r}{R'}\right) \cos \pi \frac{z}{H'}$$

HETEROGENEOUS REACTORS

- CONSIDER A HETEROGENEOUS REACTOR CONSISTING OF "N" FUEL ASSEMBLIES OF "n" RODS:
  - r_o IS THE FUEL RADIUS,
  - H LENGTH OF THE FUEL,
  - $\overline{\Sigma}_{fr}$ MACROSCOPIC CROSS SECTION OF THE FUEL.

Σ_{fr} for fuel rod

- TOTAL FISSION CROSS SECTION OF THE ENTIRE CORE:

$$\overline{\Sigma}_{fr} N n \pi r_o^2 H$$

- AVERAGE VALUE OF $\overline{\Sigma}_{fm}$ IN THE CORE IS GIVEN BY

$$\overline{\Sigma}_{fm} = \frac{\overline{\Sigma}_{fr} N n \pi r_o^2 H}{\pi R^2 H} = \frac{\overline{\Sigma}_{fr} N n r_o^2}{R^2}$$

- COMBINING $\overline{\Sigma}_{fm}$ WITH

$$\phi = \frac{2.35 \mathbf{P}_t}{E_R \overline{\Sigma}_{fm} V} J_o\left(2.405 \frac{r}{R'}\right) \cos \pi \frac{z}{H'}$$

WE OBTAIN:

$$\phi = 0.75 \frac{\mathbf{P}_t}{E_R \overline{\Sigma}_{fr} H r_o^2 N n} J_o\left(2.405 \frac{r}{R'}\right) \cos \pi \frac{z}{H'}$$

DISTRIBUTION OF HEAT SOURCES IN THE REACTOR

$$\boxed{q''' = \alpha\, E_R \overline{\Sigma}_f\, \phi(r,z)}$$

$$\boxed{\phi = 0.75 \frac{\mathbf{P}_t}{E_R \overline{\Sigma}_{fr}\, H r_o^2 N n} \times J_o\left(2.405\frac{r}{R'}\right) cos\, \pi \frac{z}{H'}}$$

$$\boxed{\overline{\Sigma}_f \Rightarrow \overline{\Sigma}_{fr}}$$

$$\boxed{q'''(r,z) = 0.75 \frac{\alpha \mathbf{P}_t}{H r_o^2 N n} J_o\left(2.405\frac{r}{R'}\right) cos\, \pi \frac{z}{H'}}$$

THIS IS THE GLOBAL POWER DENSITY DISTRIBUTION IN THE REACTOR CORE.

- THE DEPENDENCE OF q''' ON r SHOWS THE CHANGE OF POWER DENSITY FROM ROD ASSEMBLY TO ROD ASSEMBLY.
- q''' IS ASSUMED TO BE CONSTANT IN A FUEL ASSEMBLY AS WELL AS IN A FUEL ROD.
- THIS ASSUMPTION IS ACCEPTABLE FOR SMALL DIAMETER NATURAL URANIUM OR LOW ENRICHMENT FUEL RODS USED IN MOST POWER REACTORS.

THERMAL STUDY OF A REACTOR CHANNEL

- DESCRIPTION OF A FUEL CHANNEL
  - ► SINGLE ROD CONFIGURATION - WITH OR WITHOUT FINS.
  - ► MULTIROD FUEL ASSEMBLY CONFIGURATION:

    - ADVANTAGE: INCREASED HEAT TRANSFER AREA,
    - DISADVANTAGE: INCREASED NEUTRON ABSORPTION.

- POWER DENSITY DISTRIBUTION ALONG THE FUEL CHANNEL:
  - ► ACCORDING TO THE NEUTRON FLUX (POWER DENSITY) DISTRIBUTION WE HAVE SELECTED THE MAXIMUM POWER DENSITY OCCURS AT $r = 0$ & $z = 0$:

$$\boxed{q'''(r,z) = 0.75\frac{\alpha \mathbf{P}_t}{H r_o^2 Nn} J_o\left(2.405\frac{r}{R'}\right) \cos\pi\frac{z}{H'}}$$

$$\boxed{\begin{gathered} r = 0 \\ z = 0 \\ q'''_{\max} = 0.75\frac{\alpha P_t}{H r_o^2 Nn} \end{gathered}}$$

$$\boxed{q'''(r,z) = q'''_{max} J_o\left(2.405\frac{r}{R'}\right) \cos\pi\frac{z}{H'}}$$

- POWER DENSITY DISTRIBUTION ALONG A CHANNEL (AXIAL DISTRIBUTION) SITUATED AT A GIVEN RADIAL DISTANCE IS:

$$q''' = q_o''' \cos \pi \frac{z}{H'}$$

WHERE:

$$q_o''' = q_{max}''' J_o\left(2.405 \frac{r}{R'}\right)$$

- IT IS MORE CONVENIENT TO WRITE THE AXIAL DISTRIBUTION AS:

$$q''' = q_o''' \cos 2 \underbrace{\boxed{\frac{\pi H}{2H'}}}_{\beta} \frac{z}{H} = q_o''' \cos 2\beta \frac{z}{H}$$

- AVERAGE POWER DENSITY IN A FUEL CHANNEL:

$$q_{av}''' = \frac{\int_{-\frac{1}{2}H}^{\frac{1}{2}H} q_o''' \cos 2\beta \frac{z}{H}\, dz}{\int_{-\frac{1}{2}H}^{\frac{1}{2}H} dz} = q_o''' \frac{\sin \beta}{\beta}$$

- LINEAR POWER DISTRIBUTION ALONG THE FUEL CHANNEL

density (linear heat gen.)

LINEAR POWER IS DEFINED AS:

$$\boxed{q' = n\pi r_o^2 q'''}$$

MW/m.

$$\boxed{q''' = q_o'''\cos\pi\frac{z}{H'}}$$

$$\boxed{q' = n\pi r_o^2 q_o'''\cos 2\beta\frac{z}{H} = q_o'\cos 2\beta\frac{z}{H}}$$

- FUEL CHANNEL POWER

$$\boxed{\mathbf{P}_c = \int_{-\frac{1}{2}H}^{\frac{1}{2}H} q'\,dz}$$

$$\boxed{q' = q_o'\cos 2\beta\frac{z}{H}}$$

$$\boxed{\mathbf{P}_c = q_o' H\frac{\sin\beta}{\beta}}$$

$$\boxed{q_{av}' = q_o'\frac{\sin\beta}{\beta}}$$

$$\boxed{\mathbf{P}_c = Hq_{av}'}$$

BASIC EQUATIONS FOR THE THERMAL STUDY OF A FUEL CHANNEL

THESE EQUATIONS ARE:

1. MASS CONSERVATION EQUATION FOR THE COOLANT.
2. MOMENTUM CONSERVATION EQUATION FOR THE COOLANT.
3. ENERGY CONSERVATION EQUATION FOR THE COOLANT.
5. ENERGY CONSERVATION EQUATION FOR THE FUEL (HEAT CONDUCTION EQUATION).
6. ENERGY CONSERVATION EQUATION FOR THE CLADDING (HEAT CONDUCTION EQUATION).

PLUS APPROPRIATE INITIAL AND BOUNDARY CONDITIONS.

IN THE FOLLOWING DISCUSSION, IT WILL BE ASSUMED THAT:

- THE PRESSURE IS CONSTANT ALONG THE FUEL CHANNEL, AND
- THE COOLANT FLUID IS INCOMPRESSIBLE.

THEREFORE,

- MOMENTUM EQUATION FOR THE COOLANT WILL BE IGNORED, AND
- THE MASS CONSERVATION EQUATION IS GIVEN BY:

$$G = const$$

COOLANT ENERGY CONSERVATION EQUATION

REFER TO FIGURE 7.2

ENERGY CONSERVATION PRINCIPLE APPLIED TO THE CONTROL VOLUME SELECTED IN THE COOLANT:

$$\left(h+\frac{\partial h}{\partial z}dz\right)GAd\tau+\frac{\partial u}{\partial \tau}Adz\rho d\tau = hGAd\tau+q_c''sdzd\tau$$

0R

$$\rho\frac{\partial u}{\partial \tau}+G\frac{\partial h}{\partial z}=\frac{q_c''s}{A}$$

HEAT CONDUCTION EQUATION FOR FUEL:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k_f\, r\frac{\partial t}{\partial r}\right)+q'''=\frac{\partial}{\partial \tau}\left(\rho_f\, c_f\, t\right) \qquad 0\le r\le r_o$$

HEAT CONDUCTION EQUATION FOR CLADDING:

$$\frac{1}{r}k_c\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right)=\frac{\partial}{\partial \tau}(\rho_c\, c_c\, t) \qquad r_o\le r\le r_c$$

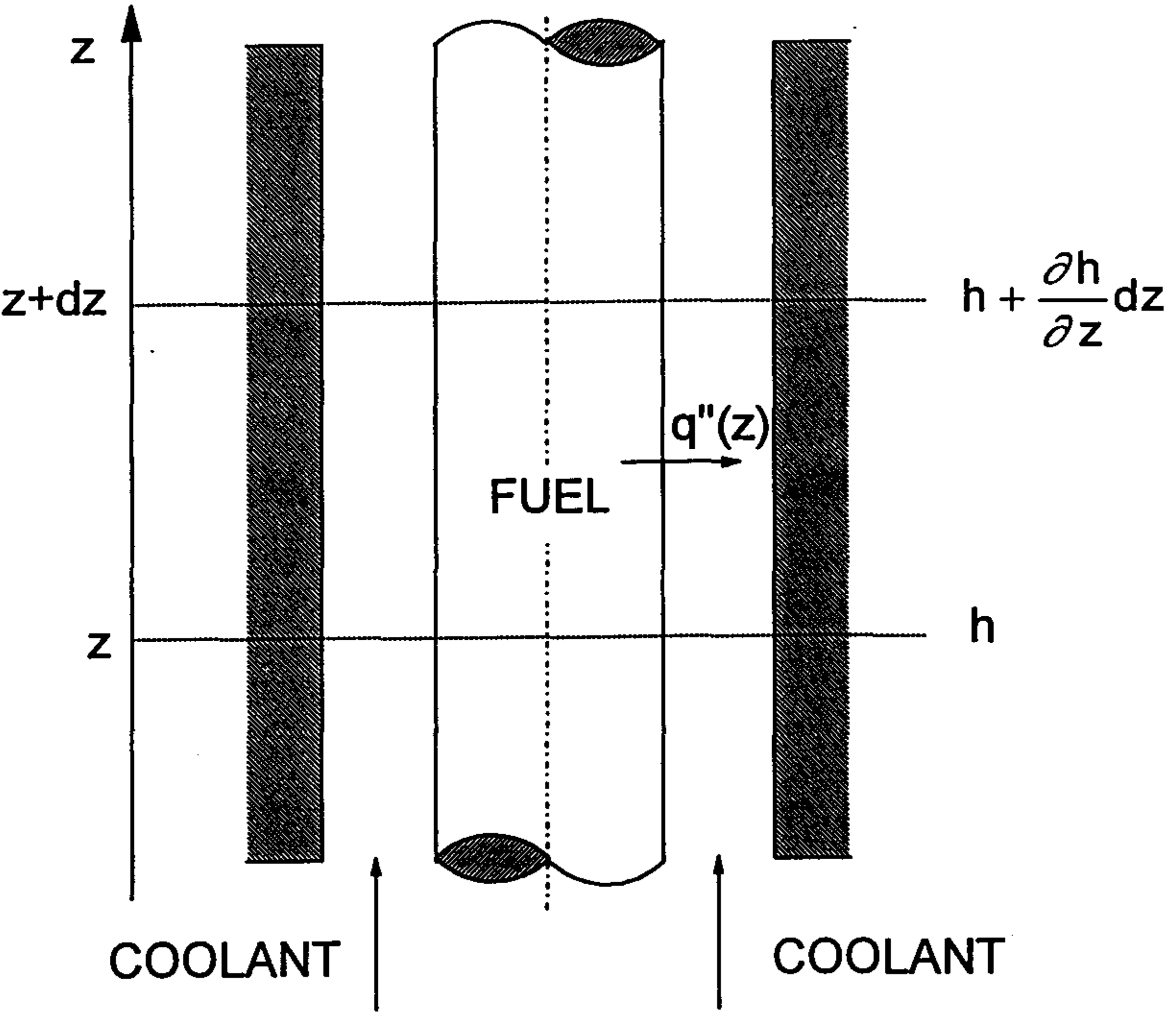

Figure 7.2 Fuel channel with single rod

BOUNDARY CONDITIONS:

- INLET MASS FLOW RATE.
- INLET TEMPERATURE.
- INLET PRESSURE.
- HEAT CONVECTION AT THE FUEL ELEMENT SURFACE:

$$q_c'' = -k_c\left(\frac{\partial t_c(r)}{\partial r}\right)_{r=r_c} = h_c\left(t_c\left(r_c\right) - t\right)$$

- HEAT TRANSFER AT THE FUEL-CLADDING INTERFACE:

$$q''_{r=r_o} = h_g\left(t_f\left(r_o\right) - t_c\left(r_o\right)\right)$$

h_g : GAP CONDUCTANCE

- CONTINUITY OF HEAT FLUXES AT THE FUEL- CLADDING INTERFACE:

$$k_f\left(\frac{\partial t_f(r)}{\partial r}\right)_{r=r_o} = k_c\left(\frac{\partial t_c(r)}{\partial r}\right)_{r=r_o}$$

THE NUMERICAL SOLUTION OF THE ENERGY EQUATIONS WITH ABOVE BOUNDARY CONDITIONS AND APPROPRIATE INITIAL CONDITIONS YIELDS:

- AXIAL DISTRIBUTION OF THE COOLANT TEMPERATURE, AND
- TEMPERATURE DISTRIBUTION IN THE FUEL AND THE CLADDING.

COOLANT TEMPERATURE VARIATION ALONG THE FUEL CHANNEL:

$$\rho \frac{\partial u}{\partial \tau} + G \frac{\partial h}{\partial z} = \frac{q_c'' s}{A}$$

STEADY STATE CONDITIONS

$$G A \frac{d h}{d z} = q_c'' s$$

$$G A = \dot{m}$$

$$q_c'' s = q'$$

$$\dot{m} \frac{d h}{d z} = q'$$

$$q' = q_o' \cos 2\beta \frac{z}{H}$$

INTEGRATION BETWEEN

$z = -\frac{1}{2} H$ and $z = z$

$$\dot{m} \int_{h_i}^{h} d h = \int_{-\frac{1}{2}H}^{z} q_o' \cos 2\beta \frac{z}{H} d z$$

FIGURE 7.3

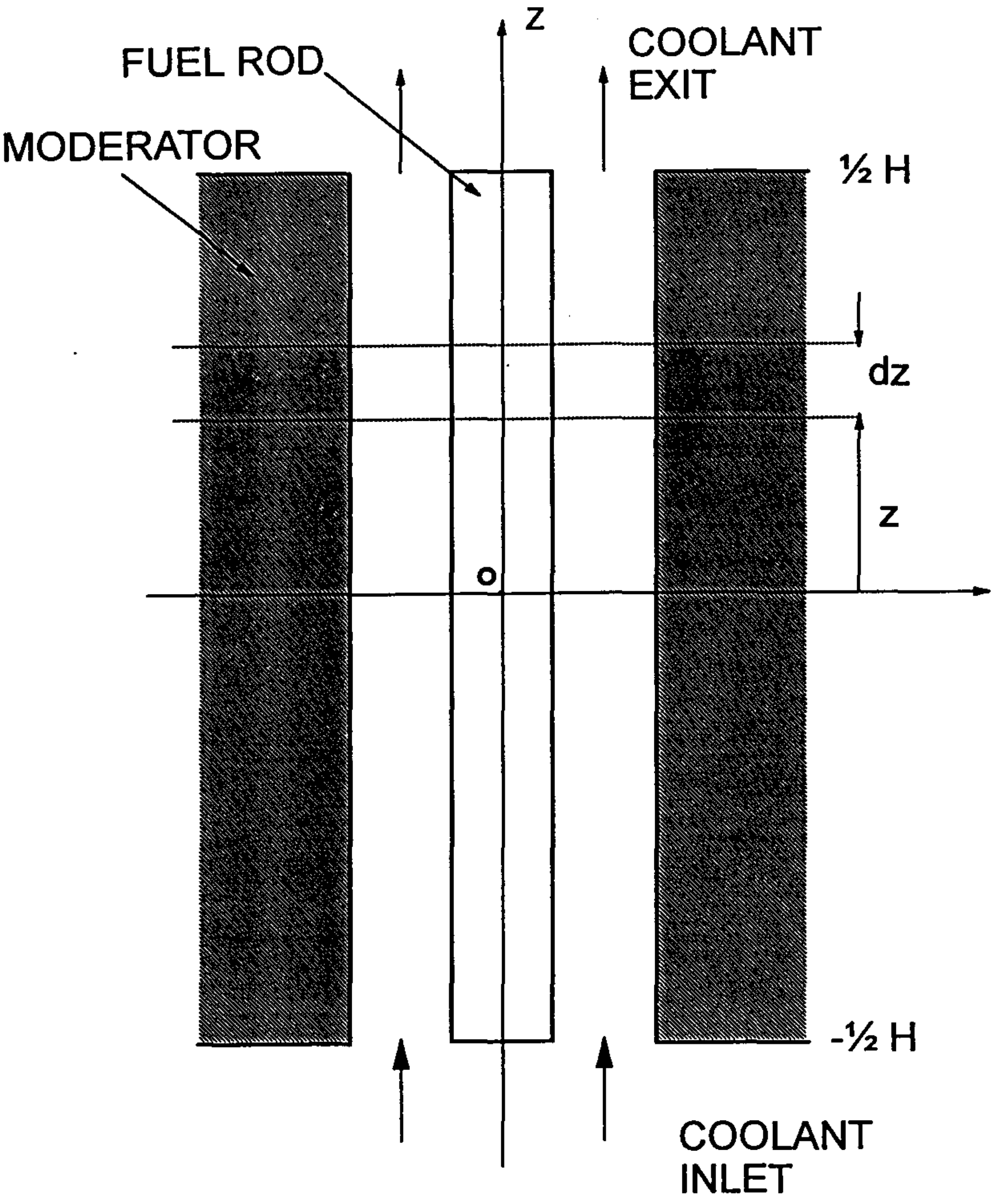

Figure 7.3 Fuel channel.

$$\dot{m}\int_{h_i}^{h} dh = \int_{-\frac{1}{2}H}^{z} q_o' \cos 2\beta \frac{z}{H}\, dz$$

$$h = h_i + \frac{q_o'}{\dot{m}}\frac{H}{2\beta}\sin\beta\left(\frac{\sin 2\beta \frac{z}{H}}{\sin\beta} + 1\right)$$

$$\mathbf{P}_c = q_o' H \frac{\sin\beta}{\beta}$$

$$h = h_i + \frac{\mathbf{P}_c}{2\dot{m}}\left(\frac{\sin 2\beta \frac{z}{H}}{\sin\beta} + 1\right)$$

$$(h - h_i) \cong \bar{c}_p (t - t_i)$$

$$t = t_i + \frac{\mathbf{P}_c}{2\dot{m}\bar{c}_p}\left(\frac{\sin 2\beta \frac{z}{H}}{\sin\beta} + 1\right)$$

$$t = t_i + \frac{\mathbf{P}_c}{2\dot{m}\overline{c}_p}\left(\frac{sin\,2\beta\,\frac{z}{H}}{sin\,\beta} + 1\right)$$

$$\mathbf{P}_c = \dot{m}\,\overline{c}_p\,(t_e - t_i)$$

$$t = \frac{t_i + t_e}{2} + \frac{t_e - t_i}{2\,sin\,\beta}\,sin\,2\beta\,\frac{z}{H}$$

$$h = h_i + \frac{\mathbf{P}_c}{2\dot{m}}\left(\frac{\sin 2\beta \frac{z}{H}}{\sin\beta} + 1\right)$$

- SUBTRACT FROM BOTH SIDES THE SATURATION ENTHALPY, h_f.
- DIVIDE BOTH SIDES BY THE LATENT HEAT OF VAPORIZATION, h_{fg}.

$$\boxed{\frac{h - h_f}{h_{fg}} = \frac{h_i - h_f}{h_{fg}} + \frac{\mathbf{P}_c}{2\dot{m} h_{fg}}\left(\frac{\sin 2\beta \frac{z}{H}}{\sin\beta} + 1\right)}$$

$$\boxed{x = \frac{h - h_f}{h_{fg}}}$$

$$\boxed{x = \frac{h_i - h_f}{h_{fg}} + \frac{\mathbf{P}_c}{2\dot{m} h_{fg}}\left(\frac{\sin 2\beta \frac{z}{H}}{\sin\beta} + 1\right)}$$

THIS IS THE VARIATION OF THE THERMODYNAMIC QUALITY, x, ALONG THE CHANNEL.

- FOR OUR APPLICATION, THE THERMODYNAMIC QUALITY MAY HAVE POSITIVE OR NEGATIVE VALUES.
- DIFFERENT VALUES OF x ARE INTERPRETED AS FOLLOWS:

$x < 0$: SUBCOOLED LIQUID

$x = 0$: SATURATED LIQUID

$0 < x < 1$: SATURATED STEAM-WATER MIXTURE

$x = 1$: SATURATED STEAM

$x > 1$: SUPERHEATED STEAM

THERMAL STUDY OF THE FUEL ROD

- TEMPERATURE OF THE CLADDING
  - MATERIALS: ZIRCALOY-4 & STAINLESS STEEL.
  - STAINLESS STEEL HAS HIGH NEUTRON ABSORPTION CROSS SECTION - REQUIRES ENRICHED URANIUM.
  - ZIRCALOY HAS LOW NEUTRON ABSORPTION CROSS SECTION - SUITABLE FOR BOTH PWR & PHWR (CANDU).
  - TEMPERATURE OF THE OUTSIDE SURFACE OF THE CLADDING:

TABLE 7.4

$$t_c - t = \frac{q'}{h_c s}$$

$$t = \frac{t_i + t_e}{2} + \frac{t_e - t_i}{2\sin\beta}\sin 2\beta\frac{z}{H}$$

$$q' = q'_o \cos 2\beta\frac{z}{H}$$

$$q'_{av} = q'_o \frac{\sin\beta}{\beta}$$

$$\mathbf{P}_c = H q'_{av}$$

$$t_c = \frac{t_i + t_e}{2} + \frac{t_e - t_i}{2\sin\beta}\sin 2\beta\frac{z}{H} + \frac{1}{h_c s}\frac{\mathbf{P}_c}{H}\frac{\beta}{\sin\beta}\cos 2\beta\frac{z}{H}$$

$$t_c = \frac{t_i + t_e}{2} + \frac{t_e - t_i}{2\,sin\beta} sin 2\beta \frac{z}{H} + \frac{1}{h_c s} \frac{\mathbf{P}_c}{H} \frac{\beta}{sin\beta} cos 2\beta \frac{z}{H}$$

$$\mathbf{P}_c = \dot{m}\,\bar{c}_p (t_e - t_i)$$

DEFINING:

$$\gamma = \frac{h_c\, s\, H}{2\beta\, \dot{m}\,\bar{c}_p}$$

$$t_c = \frac{t_i + t_e}{2} + \frac{t_e - t_i}{2\,sin\beta}\left(sin 2\beta \frac{z}{H} + \frac{1}{\gamma} cos 2\beta \frac{z}{H} \right)$$

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THIS EQUATION GIVES THE VARIATION OF THE OUTSIDE TEMPERATURE OF THE CLADDING ALONG THE FUEL CHANNEL.

Table 7.4 Thermal properties of Zircaloy-4

Table 7.1 Thermal Properties of Zircaloy-4

Density	Conductivity	Specific heat	Melting point	Limiting temperature
kg/m^3	W/mK (at °C)	J/kgK (at °C)	°C	°C
6570	12.7 (300)	328 (300)	1850	40
	13.1 (400)	357 (650)		

MAXIMUM CLADDING TEMPERATURE:

$$t_c = \frac{t_i + t_e}{2} + \frac{t_e - t_i}{2\,sin\,\beta}\left(sin\,2\beta\frac{z}{H} + \frac{1}{\gamma}cos\,2\beta\frac{z}{H}\right)$$

DERIVATION

$$\frac{d\,t_c}{d\,z} = cos\,2\beta\frac{z}{H} - \frac{1}{\gamma}sin\,2\beta\frac{z}{H} = 0$$

$$or \qquad \gamma = tan\,2\beta\frac{z}{H}$$

THE MAXIMUM TEMPERATURE IS LOCATED AT: $z_m = \frac{H}{2\beta}arctan\,\gamma$

THE VALUE OF THE MAXIMUM TEMPERATURE

$$t_c = \frac{t_i + t_e}{2} + \frac{t_e - t_i}{2\,sin\,\beta}\left(sin\,2\beta\frac{z}{H} + \frac{1}{\gamma}cos\,2\beta\frac{z}{H}\right)$$

$$z_m = \frac{H}{2\beta}arctan\,\gamma$$

$$t_{cmax} = \frac{t_i + t_e}{2} + \frac{t_e - t_i}{2\,sin\,\beta}\left[sin(arctan\,\gamma) + \frac{1}{\gamma}cos(arctan\,\gamma)\right]$$

MAXIMUM CLADDING TEMPERATURE SHOULD BE LESS THEN 380 - 400 °C.

NORMAL OPERATING TEMPERATURE OF THE CLADDING IS ABOUT 320 °C.

- TEMPERATURE DISTRIBUTION IN THE CLADDING IN THE RADIAL DIRECTION AT A GIVEN AXIAL LOCATION $(k_c = const.)$.

$$\frac{1}{r} k_c \frac{\partial}{\partial r}\left(r \frac{\partial t}{\partial r} \right) = \frac{\partial}{\partial \tau}(\rho_c c_c t) \qquad r_o \le r \le r_c$$

STEADY STATE CONDITION

$$\frac{1}{r} k_c \frac{\partial}{\partial r}\left(r \frac{\partial t}{\partial r} \right) = 0 \qquad r_o \le r \le r_c$$

SOLUTION $\Rightarrow$ $t = A \ln r + B$

BOUNDARY CONDITIONS

$$r = r_o \quad q' = -2\pi r_o k_c \frac{d t(r)}{dr} \qquad \Rightarrow \qquad A = -\frac{q'}{2\pi k_c}$$

$$r = r_c \quad t(r) = t_c \qquad \Rightarrow \qquad B = t_c + \frac{q'}{2\pi k_c} ln\, r_c$$

VARIATION OF THE TEMPERATURE THROUGH THE CLADDING :

$$t = t_c + \frac{q'}{2\pi k_c} ln\frac{r_c}{r} \quad with \quad r_o \leq r \leq r_c$$

TEMPERATURE OF THE INNER SURFACE OF THE CLADDING $(r = r_o)$:

$$t_c' = t_c + \frac{q'}{2\pi k_c} \ln\frac{r_c}{r_o}$$

- GAP CONDUCTANCE BETWEEN FUEL AND CLADDING

  - ► BECAUSE THE LOW HEAT CONDUCTANCE, A LARGE TEMPERATURE DROP OCCURS IN THE GAP REGION (AT THE INTERFACE) BETWEEN THE FUEL AND CLADDING.

  - ► THIS DROP (~ 150 °C) IS SECOND TO THAT IN THE CERAMIC FUEL (URANIUM DIOXIDE).

  - ► DURING THE MANUFACTURING OF THE FUEL RODS, A GAP OF ABOUT 0.08mm IS REQUIRED TO INSERT FUEL PALLETS INTO CLADDING TUBES.

  - ► GRAPHITE POWDER IS ALSO USED (CANDU REACTORS) TO EASE THE INSERTION OF THE PALLETS.

  - ► FUEL PALLETS ARE LOADED IN AN ATMOSPHERE OF AN INERT GAS (e.g. HELIUM).

    1. TO AVOID THE CORROSION, AND

    2. TO ASSURE A REASONABLE INITIAL HEAT TRANSFER COEFFICIENT.

GAP CONDUCTANCE BETWEEN FUEL AND CLADDING

- THE GAP REGION IS SO NARROW THAT THERE IS NO CONVECTION CURRENTS IN THE GAS.
- CONSEQUENTLY, AT THE EARLY STAGE, HEAT TRANSFER IN THE GAP REGION IS , ESSENTIALLY, BY CONDUCTION THROUGH THE FILLING GAS (SOME RADIATION HEAT TRANSFER IS ALSO PRESENT)
- LATTER, BECAUSE OF THE SWELLING AND THERMAL EXPANSION OF THE FUEL WHICH IS MORE IMPORTANT TO THAT OF THE CLADDING, THE GAP REGION CLOSES.
- CONSEQUENTLY, DIRECT CONTACT TAKES PLACE BETWEEN SURFACES AT SEVERAL DISCRETE POINTS.
- THE HEAT TRANSFER IS THEN:
  1. BY CONDUCTION AT THESE SOLID-TO-SOLID CONTACT POINTS, AND ALSO
  2. BY CONDUCTION THROUGH THE FILLING GAS.

- THE GAP HEAT TRANSFER IS USUALLY EXPRESSED AS:

$$q' = 2\pi r_o h_g \left(t_f - t_c'\right)$$

WHERE h_g IS THE GAP CONDUCTANCE.

- BASED ON THE ABOVE DISCUSSION, THE GAP BETWEEN THE FUEL AND CLADDING IS CLASSIFIED AS:

1. OPEN GAP
2. CLOSED GAP

1. OPEN GAP

- THE FUEL AND CLADDING ARE NOT IN PHYSICAL CONTACT.
- TRUE FOR FRESH FUEL.
- THE FUEL STANDS FREELY WITHIN THE CLADDING.
- HEAT TRANSFER MECHANISMS ARE:

1. CONDUCTION THROUGH THE FILLING GAS, AND
2. RADIATION.

- INITIALLY THE FILLING GAS IS PURE HELIUM.

- LATTER ON FISSION GASES (KRYPTON & XENON) FILL THE GAP REGION AND MIX WITH HELIUM.

- THIS ACCUMULATION INCREASES THE PRESSURE IN THE GAP. I.E., THE PRESSURE IN THE FUEL ELEMENT:

- TOWARD THE END OF FUEL LIFE

- THE PRESSURE IS ~ 7.5 MPa
- THE COMPOSITION OF THE FILLING GAS: HELIUM, KRYPTON (~15%), XENON (~85%)

- IF THE GAP CLEARANCE IS MUCH LARGER THEN THE **MEAN FREE PATH OF THE ATOMS**, THE GAP CONDUCTANCE IS GIVEN BY:

$$h_g = \frac{k_g}{\delta_g} + \frac{\sigma}{\frac{1}{\varepsilon_f} + \frac{1}{\varepsilon_c} - 1} \frac{T_f^4 - T_c'^4}{T_f - T_c'}$$

- THE VARIATION OF THE GAS TEMPERATURE IN THE GAP REGION IS SHOWN IN:

FIGURE 7.4

- A STEEP CHANGE IN TEMPERATURES TAKES PLACE IN THE REGIONS CLOSE TO THE SOLID SURFACES.

- THE EXTEND OF THE REGION IS ABOUT A **MEAN FREE PATH.**

- THIS STEEP CHANGE IS CALLED **"TEMPERATURE JUMP."**

- TO APPLY PROPERLY THE HEAT CONDUCTION LAW, THE LINEAR PORTION OF THE TEMPERATURE PROFILE SHOULD BE EXTRAPOLATED UNTIL IT CROSSES THE FUEL AND CLADDING TEMPERATURES AT δ_f *and* δ_c .

- δ_f *and* δ_c ARE CALLED **"TEMPERATURE JUMP DISTANCES"**

- THESE DISTANCES SHOULD BE ADDED TO THE ACTUAL GAP THICKNESS TO PREDICT THE CORRECT FUEL-CLADDING TEMPERATURE DIFFERENCE BY USING THE FOURIER CONDUCTION LAW.

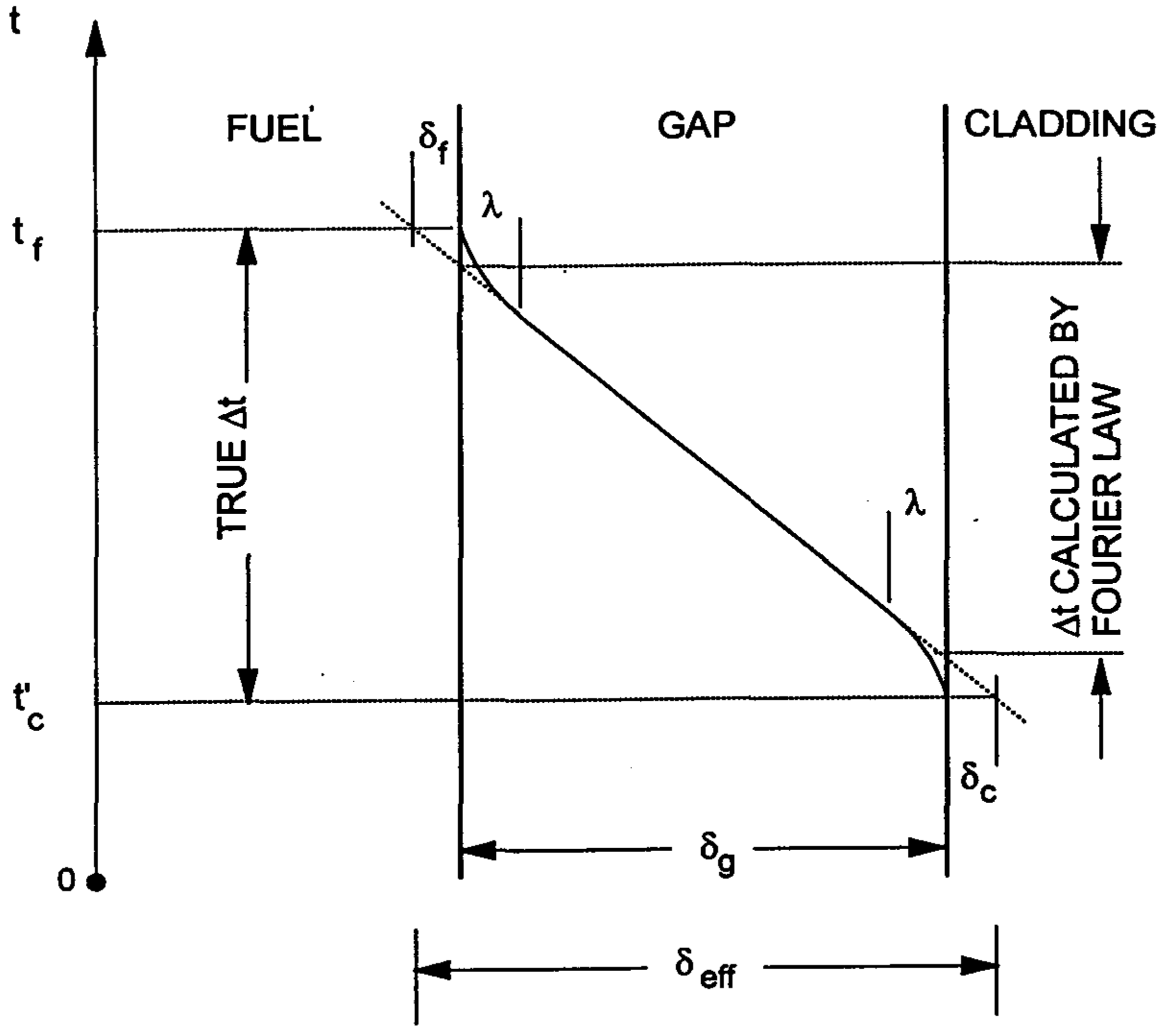


Figure 7.4 Temperature profile in the gap region and temperature jump

- THE CONDUCTANCE OF A NARROW GAP (FEW MEAN FREE PATH) IS GIVEN BY:

$$h_g = \frac{k_g}{\delta_g + \delta_f + \delta_c} + \frac{\sigma}{\dfrac{1}{\varepsilon_f} + \dfrac{1}{\varepsilon_c} - 1} \frac{T_f^4 - T_c'^4}{T_f - T_c'}$$

- KENNARD GIVES THE TEMPERATURE JUMP DISTANCE AS:

$$\delta = 2\left(\frac{2-\alpha}{\alpha}\right)\left(\frac{\Gamma}{1-\Gamma}\right)\left(\frac{k}{\mu\, c_p}\right)\lambda$$

λ : MEAN FREE PATH,

Γ : RATIO OF c_p / c_v FOR THE GASES,

c_p : SPECIFIC HEAT AT CONSTANT PRESSURE,

c_v : SPECIFIC HEAT AT CONSTANT VOLUME,

μ : VISCOSITY OF THE GAS,

k : CONDUCTIVITY OF THE: solid

- α IS THE ACCOMMODATION COEFFICIENT OF THE GAS IN CONTACT WITH A SOLID.

- IT TAKES INTO ACCOUNT THE INCOMPLETE ENERGY INTERCHANGE BETWEEN THE GAS MOLECULES AND THE SOLID SURFACE.

- THE ACCOMMODATION COEFFICIENT IS GIVEN BY:

$$\alpha = \frac{T_r - T_i}{T_s - T_i}$$

T_i :TEMPERATURE OF THE MOLECULES THAT STRIKE THE SOLID,

T_s :TEMPERATURE OF THE SOLID,

T_r :TEMPERATURE OF THE REFLECTED MOLECULES.

- α IS LESS THAN 1.
- $\alpha = 1$, REFLECTED MOLECULES ARE IN EQUILIBRIUM WITH SOLID.
- SURFACE ROUGHNESS INCREASES THE VALUE OF THE THERMAL ACCOMMODATION COEFFICIENT BY MAKING MULTIPLE COLLISIONS POSSIBLE BETWEEN THE MOLECULES AND THE SOLID SURFACE.

- THE MEAN FREE PATH, λ , IS GIVEN BY:

$$\lambda = \frac{\lambda_o}{p}\frac{T}{273}$$

T : ABSOLUTE TEMPERATURE (K),

p : PRESSURE (BAR),

λ_o : PROPERTY OF THE GAS.

$$helium \quad \lambda_o = 1.74 \times 10^{-5} \ bar - cm$$

$$xenon \quad \lambda_o = 3.60 \times 10^{-6} \ bar - cm$$

- MEAN FREE PATH FOR HELIUM AT 1 BAR AND 293 K:

$$\lambda = 0.2\ \mu m$$

- MEAN FREE PATH FOR XENON AT 10 BARS AND 1000 K:

$$\lambda = 0.01\ \mu m$$

► THE RADIATION COMPONENT OF THE GAP CONDUCTANCE IS USUALLY SMALL UNDER NORMAL OPERATING CONDITIONS, THEREFORE:

$$h_g = \frac{k_g}{\delta_g + \delta_f + \delta_c}$$

- THE GAP WIDTH IS TYPICALLY 80 μm

- δ_f, δ_c ARE IN THE ORDER OF MAGNITUDE OF MEAN FREE PATH; THEREFORE THEY CAN BE NEGLECTED.

- THIS IS TRUE UNTIL THE GAP CLOSES AND THE SOLIDS MAKE CONTACT.

► CONDUCTIVITY OF THE FILLING GAS IS GIVEN BY:

$$k_g = A \cdot 10^{-6}\, T^{0.79} \quad W / cm\,K$$

A IS A CONSTANT.

► CONDUCTIVITY FOR A MIXTURE OF TWO GASES IS GIVEN BY:

$$k_g = (k_1)^{x_1} (k_2)^{x_2}$$

k_1, k_2 : CONDUCTIVITY OF THE GASES

x_1, x_2 : MOLE FRACTIONS

2. CLOSED GAP

- BECAUSE OF:
  - THE FUEL SWELLING,
  - THE DIFFERENTIAL EXPANSION OF THE FUEL, AND
  - PRESSURE EXERTED BY THE COOLANT ON THE OUTSIDE SURFACE OF THE CLADDING

  THE GAP TENDS TO CLOSE

- IN TURN

  - PRESSURE BUILD-UP IN THE GAP REGION DUE TO THE ACCUMULATION OF THE FISSION GASES, AND
  - FUEL DIAMETER REDUCTION DUE TO THE FUEL DENSIFICATION

  THE GAP TENDS TO OPEN.

- THE EFFECTS THAT CLOSE THE GAP ARE MORE IMPORTANT THAN THOSE THAT OPEN THE GAP.

- CONSEQUENTLY THE GAP THICKNESS IS CONSIDERABLY REDUCED.

gap
clearan

- BECAUSE OF THE ROUGHNESS OF THE SURFACES, SOLID-TO-SOLID CONTACT BETWEEN THE FUEL AND CLADDING IS ESTABLISHED AT HIGH POINTS.

- UNDER THIS CONDITION, HEAT TRANSFER IN THE GAP REGION OCCURS:

  1. THROUGH THE POINTS OF SOLID CONTACT,

  2. ACROSS THE NOW DISCONTINUOUS GAS GAP BETWEEN THESE POINTS OF CONTACT.

REFER TO FIGURE 7.5

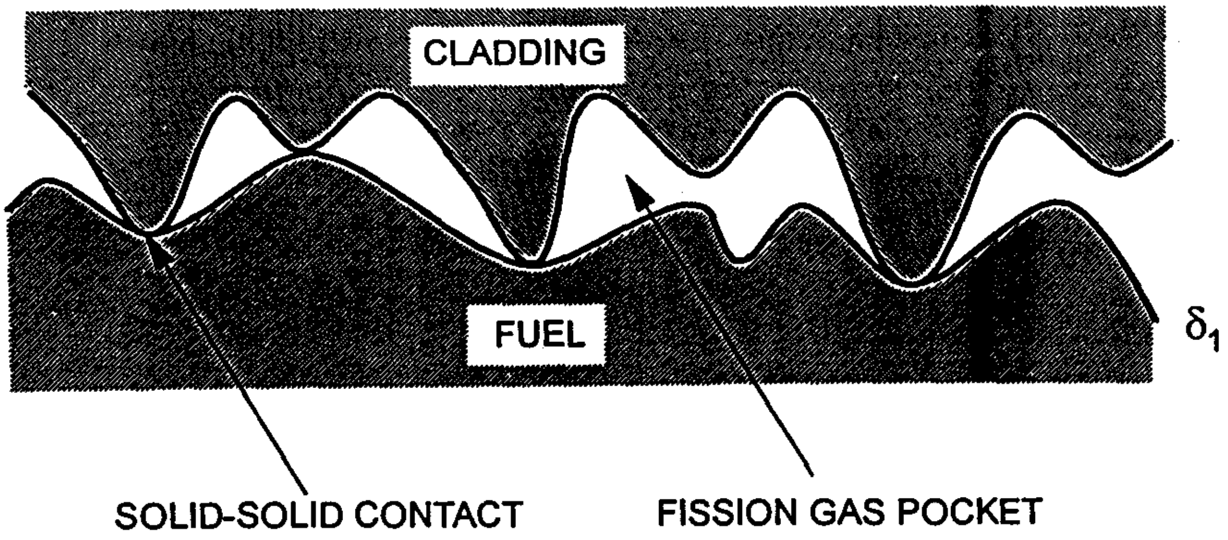


Figure 7.5 Closed fuel cladding gap

- FOR A CLOSED GAP, DIFFERENT COMPONENTS OF THE CONDUCTANCE ARE:

  1. GAS CONDUCTANCE THROUGH DISCONTINUOUS GAP,
  2. CONDUCTION THROUGH THE SOLID-TO-SOLID CONTACT POINTS,
  3. RADIATION THROUGH THE DISCONTINUOUS GAP.

- CONDUCTANCE DUE TO SOLID-TO-SOLID CONTACT IS GIVEN BY:

$$h_s = A \frac{2k_f\, k_c}{k_f + k_c} \frac{p_i}{H\,\delta^{1/2}}$$

A : DIMENSIONAL CONSTANT, $m^{1/2}$ $\left(= 10\ m^{1/2}\right)$

k_f : CONDUCTIVITY OF THE FUEL, $W/m\ {}^{o}C$

k_c : CONDUCTIVITY OF THE CLADDING, $W/m\ {}^{o}C$

p_i : SURFACE CONTACT PRESSURE, N/m^2

H : MEYER HARDNESS NUMBER OF THE SOFTER MATERIAL, N/m^2

δ : RMS OF CONTACT MATERIAL SURFACE ROUGHNESS AND GIVEN BY:

$$\delta = \left[\left(\delta_1^2 + \delta_2^2\right)/2\right]^{1/2} \quad m$$

δ_1, δ_2 : SURFACE ROUGHNESS OF THE INTERFACE MATERIAL, m.

- GAP CONDUCTANCE THROUGH DISCONTINUOUS GAP WAS GIVEN BY:

$$h_g = \frac{k_g}{\delta_g + \delta_f + \delta_c}$$

- THE THICKNESS OF A CLOSED GAP IS RELATED TO THE SURFACE ~~THICKNESS~~ ROUGHNESS AS:

$$\delta_g = C(\delta_1 + \delta_2)$$

C IS A CONSTANT GIVEN BY:

$$C = 2.75 - 2.55 \times 10^{-8} p_i$$

p_i IS THE PRESSURE AT THE INTERFACE IN N/m^2.

- TAKING INTO CONSIDERATION THE VALUE OF δ_g THE GAP CONDUCTANCE BECOMES:

$$h_g = \frac{k_g}{C(\delta_1 + \delta_2) + \delta_f + \delta_c}$$

TEMPERATURE JUMP DISTANCES FOR VARIOUS GASES

Gas	$\delta_f + \delta_c$, m	Temperature range, °C
Helium	10×10^{-6}	150-250
Argon	5×10^{-6}	180-320
Krypton	1×10^{-6}	180-330
Xenon	1×10^{-6}	180-330

- THE TOTAL CONDUCTANCE OF A CLOSED GAP IS GIVEN BY:

$$\begin{aligned} h_{con} &= h_s + h_g + h_r \\ &= A\frac{2k_f k_c}{k_f + k_c} \times \frac{p_i}{H\delta^{1/2}} + \frac{k_g}{C(\delta_1 + \delta_2) + \delta_f + \delta_c} \\ &+ \frac{\sigma}{\frac{1}{\varepsilon_f} + \frac{1}{\varepsilon_c} - 1} \times \frac{T_f^4 - T_c'^4}{T_f - T_c'} \end{aligned}$$

- SURFACE TEMPERATURE OF THE FUEL

$$q' = 2\pi r_o h_{con}\left(t_f - t'_c\right)$$

$$t'_c = t_c + \frac{q'}{2\pi k_c}\ln\frac{r_c}{r_o}$$

$$t_f = t_c + \frac{q'}{2\pi k_c}\ln\frac{r_c}{r_o} + \frac{q'}{2\pi r_o h_{con}}$$

$$t_c = \frac{t_i + t_e}{2} + \frac{t_e - t_i}{2\,sin\beta}\left(sin\,2\beta\frac{z}{H} + \frac{1}{\gamma}cos\,2\beta\frac{z}{H}\right)$$

$$q' = q'_o\,cos\,2\beta\frac{z}{H}$$

$$\mathbf{P}_c = q'_o H\frac{sin\beta}{\beta}$$

$$t_f = \frac{t_i + t_e}{2} + \frac{t_e - t_i}{2\,sin\beta}\left[sin\,2\beta\frac{z}{H} + \left(\frac{1}{\gamma} + \frac{\beta}{\pi k_c}\frac{\mathbf{P}_c}{(t_e - t_i)H}ln\frac{r_c}{r_o} + \frac{\beta}{\pi r_o h_{con}}\frac{\mathbf{P}_c}{(t_e - t_i)H}\right)cos\,2\beta\frac{z}{H}\right]$$

SURFACE TEMPERATURE OF THE FUEL

$$\frac{1}{\gamma'}=\left(\frac{1}{\gamma}+\frac{\beta}{\pi\, k_c}\frac{\mathbf{P}_c}{(t_e-t_i)H}\,ln\frac{r_c}{r_o}+\frac{\beta}{\pi\, r_o h_{con}}\frac{P_c}{(t_e-t_i)H}\right)$$

$$t_f=\frac{t_i+t_e}{2}+\frac{t_e-t_i}{2\,sin\beta}+\left(sin\,2\beta\frac{z}{H}+\frac{1}{\gamma'}cos\,2\beta\frac{z}{H}\right)$$

- THE MAXIMUM TEMPERATURE OCCURS AT:

$$z_m=\frac{H}{2\beta}arctan\gamma'$$

- THE VALUE OF THE MAXIMUM TEMPERATURE OF THE FUEL SURFACE IS:

$$t_{f\,max}=\frac{t_i+t_e}{2}+\frac{t_e-t_i}{2\,sin\beta}\sqrt{1+\left(\frac{1}{\gamma'}\right)^2}$$

- TEMPERATURE DISTRIBUTION IN THE FUEL

NOW WE WILL DISCUSS:

1. THE DISTRIBUTION OF RADIAL TEMPERATURES IN THE FUEL, AND
2. THE VARIATION OF THE FUEL CENTERLINE TEMPERATURE ALONG THE CHANNEL.

IN THE DETERMINATION OF THE RADIAL TEMPERATURE DISTRIBUTION THE FOLLOWING ASSUMPTIONS WILL BE MADE:

1. UNIFORM NEUTRON FLUX (POWER DENSITY) WITHIN THE FUEL.
2. NO ANGULAR VARIATION IN THE CONVECTIVE HEAT TRANSFER COEFFICIENT AND GAP CONDUCTANCE, I.E., NO ANGULAR TEMPERATURE GRADIENT.
3. AXIAL HEAT CONDUCTION IS SMALL COMPARED TO THAT IN THE RADIAL DIRECTION.
4. STEADY STATE CONDITIONS PREVAIL.

► UNDER ABOVE CONDITIONS, HEAT CONDUCTION EQUATION:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k_f\, r\frac{\partial t}{\partial r}\right)+q'''=\frac{\partial}{\partial \tau}\left(\rho_f\, c_f\, t\right) \qquad 0\le r\le r_o$$

BECOMES:

$$\frac{1}{r}\left(\frac{d}{dr}k_f\, r\frac{dt}{dr}\right)+q'''=0$$

TEMPERATURE DISTRIBUTION IN THE FUEL

$$\frac{1}{r}\left(\frac{d}{d\,r}k_f\,r\frac{d\,t}{d\,r}\right)+q'''=0$$

INTEGRATING ONCE

$$k_f\frac{d\,t}{d\,r}+q'''\frac{r}{2}+\frac{A}{r}=0$$

ASSUME THAT THE FUEL HAS AN INTERNAL CAVITY OF RADIUS r_i WHICH IS NOT COOLED. THE CORRESPONDING BOUNDARY CONDITION IS:

$$q''_{r=r_i}=-k_f\left(\frac{d\,t}{d\,r}\right)_{r=r_i}=0$$

THE CONSTANT A IS THEN GIVEN BY:

$$A=-\frac{q'''r_i^2}{2}$$

INTEGRATING A SECOND TIME

$$\int_t^{t_f}k_f\,d\,t=-\frac{q'''}{4}\left(r_o^2-r^2\right)+\frac{q'''}{2}r_i^2\,ln\frac{r_o}{r}\qquad or$$

$$\int_{t_f}^{t}k_f\,d\,t=\frac{q'''r_o^2}{4}\left[1-\left(\frac{r}{r_o}\right)^2-\left(\frac{r_i}{r_o}\right)^2 ln\left(\frac{r_o}{r}\right)^2\right]$$

TEMPERATURE DISTRIBUTION IN THE FUEL - INTEGRAL OF CONDUCTIVITY

IF $r_i = 0$

$$\int_{t_s}^{t} k_f \, dt = \frac{q''' r_o^2}{4}\left[1-\left(\frac{r}{r_o}\right)^2\right]$$

IF $r = 0$

$t = t_o$

$$\int_{t_s}^{t_o} k_f \, dt = \frac{q''' r_o^2}{4}$$

$q' = \pi r_o^2 q'''$

$$\int_{t_s}^{t_o} k_f \, dt = \frac{q'}{4\pi}$$

$\int_{t_s}^{t_o} k_f \, dt$ IS CALLED INTEGRAL OF CONDUCTIVITY.

IT IS INTERESTING TO NOTE THAT:

- THE TEMPERATURE DIFFERENCE ACROSS THE FUEL IS FIXED BY **THE LINEAR HEAT FLUX.**
- A LIMIT ON THE **LINEAR HEAT FLUX** IS IMPOSED TO RESPECT THE MAXIMUM TEMPERATURE.

TEMPERATURE DISTRIBUTION IN THE FUEL - INTEGRAL OF CONDUCTIVITY

$$\boxed{\int_{t_s}^{t_o} k_f \, dt = \frac{q'}{4\pi}}$$

PROPERTIES OF "THE CONDUCTIVITY INTEGRAL:"

1. IT IS DIRECTLY RELATED TO THE LINEAR POWER OF THE FUEL.
2. IT CAN BE DETERMINED EXPERIMENTALLY.
3. CENTER LINE TEMPERATURE OF THE FUEL t_o IS INDEPENDENT OF THE ROD DIAMETER.
4. THE UPPER LIMIT OF THE INTEGRAL CAN BE TAKEN AS A VARIABLE - THE DIFFERENTIATION YIELDS THE THERMAL CONDUCTIVITY.
5. THE CONDUCTIVITY INTEGRAL CAN BE WRITTEN AS:

$$\boxed{\int_{t_s}^{t_o} k_f \, dt = \int_{0}^{t_o} k_f \, dt - \int_{0}^{t_s} k_f \, dt}$$

   I.E., NORMALIZED TO 0 °C.

6. THE CONDUCTIVITY INTEGRAL CAN BE USED TO DETERMINE CENTERLINE TEMPERATURE OF THE FUEL.

1. METALLIC FUELS

- URANIUM METAL IS A VERY POOR FUEL.

- SUBSTANTIAL DIMENSIONAL CHANGES UNDER IRRADIATION AND THERMAL CYCLING (UP TO 60% OF THE ORIGINAL LENGTH).

- THIS CHANGE IS DUE TO "α-PHASE" PRESENT UNDER 665 °C.

- "α-PHASE" IS ANISOTROPIC.

- CONSEQUENTLY, FUEL ROD:

  EXPANDS IN THE AXIAL DIRECTION.
  CONTRACTS IN THE RADIAL DIRECTION.

- CONSEQUENCE:

  CLADDING FAILURE, AND
  EXPOSITION OF THE FUEL TO THE COOLANT.

- STRONG CHEMICAL REACTION BETWEEN FUEL AND COOLANT.

- AT 290 °C, EXPOSED FUEL IS DESTROYED IN FEW HOURS.

- ALLOYING MATERIALS (Zr, Cr, Mo, Fe, etc.) IMPROVE THE DIMENSIONAL STABILITY BUT THEY INCREASE PARASITIC NEUTRON ABSORPTION.

TEMPERATURE DISTRIBUTION IN THE FUEL - FUEL MATERIALS-METALLIC

- URANIUM METAL IS UNFIT FOR USE IN **BWR, PWR, CANDU** REACTORS.

- CONDUCTIVITY OF THE URANIUM METAL VARIES MODERATELY WITH TEMPERATURE.

- THE CONDUCTIVITY OF A PURE URANIUM METAL AS A FUNCTION OF TEMPERATURE IS GIVEN BY:

$$k_f = 25.58 + 1.75\left(\frac{t}{100}\right) + 0.018\left(\frac{t}{100}\right)^3 \quad W/m\ {}^{\circ}C$$

$$t \quad in \quad {}^{\circ}C$$

- IN THE DETERMINATION OF THE FUEL CENTERLINE TEMPERATURE, THE CONDUCTIVITY OF THE URANIUM METAL CAN BE TAKEN AS CONSTANT.

SEE

Table 7.6 Conductivity of fuel materials

Table 7.6 Conductivity of fuel materials

Température	Uranium	UO_2	Thorium	ThO_2	UC
°F	K_c = Btu/hr pd °F				
200	15.80	4.5	21.75	7.29	14.77
300	16.40	-	22.18	6.25	14.07
400	17.00	3.5	22.60	5.34	13.48
500	17.50	-	23.00	4.61	13.02
600	18.10	2.8	23.45	4.03	12.67
700	18.62	-	23.90	3.59	12.39
800	19.20	2.5	24.30	3.21	12.19
900	19.70	-	24.65	2.91	12.02
1 000	20.25	2.2	25.75	2.68	11.91
1 100	20.75	-	25.60	2.47	
1 200	21.20	2.0	26.13	2.30	11.82
1 300	21.60	-	-	2.17	
1 400	22.00	1.6	-	2.07	11.76
1 600	-	1.5	-	1.90	11.70
1 800	-	1.4	-	1.80	11.67
2 000	-	1.3	-	1.70	11.57
2 200	-	1.2	-	1.69	-
2 400	-	1.1	-	1.68	-
2 600	-	1.1	-	-	-
3 200	-	1.1	-	-	-

CENTERLINE TEMPERATURE OF THE FUEL ALONG THE CHANNEL

$$\int_{t_f}^{t} k_f\, d\,t = \frac{q'''r_o^2}{4}\left[1-\left(\frac{r}{r_o}\right)^2\right]$$

k_f CONSTANT

$$t = t_f + \frac{q'}{4\,\pi\, k_f}\left[1-\left(\frac{r}{r_o}\right)^2\right]$$

$$q' = q'_o \cos 2\beta \frac{z}{H}$$

$$\mathbf{P}_c = q'_o H \frac{\sin\beta}{\beta}$$

$$t_f = \frac{t_i + t_e}{2} + \frac{t_e - t_i}{2\sin\beta} + \left(\begin{array}{l} \sin 2\beta \dfrac{z}{H} \\ + \dfrac{1}{\gamma'} \cos 2\beta \dfrac{z}{H} \end{array}\right)$$

CENTERLINE TEMPERATURE OF THE FUEL ALONG THE CHANNEL

$$\boxed{t_o = \frac{t_i + t_e}{2} + \frac{t_e - t_i}{2\,sin\,\beta}\left(sin\,2\beta\frac{z}{H} + \frac{1}{\gamma''}cos\,2\beta\frac{z}{H}\right)}$$

WHERE:

$$\frac{1}{\gamma''} = \frac{1}{\gamma'} + \frac{\beta}{2\pi\,k_f}\frac{\mathbf{P_c}}{(t_e - t_i)H}$$

MAXIMUM CENTERLINE TEMPERATURE IS LOCATED AT:

$$z = \frac{H}{2\beta}arctan\,\gamma''$$

MAXIMUM CENTERLINE TEMPERATURE IS:

$$t_{o\,max} = \frac{t_i + t_e}{2\,sin\,\beta} + \frac{t_e - t_i}{2\,sin\,\beta}\sqrt{1 + \left(\frac{1}{\gamma''}\right)^2}$$

II. CERAMIC FUELS

- URANIUM DIOXIDE - A CERAMIC MATERIAL IS WIDELY USED IN POWER REACTORS.

- ADVANTAGES:

  - ► DIMENSIONAL STABILITY.
  - ► ADEQUATE RESISTANCE TO RADIATION.
  - ► HIGH MELTING POINT: ~ 2800 ° C.
  - ► NO CHEMICAL REACTION WITH HOT LIGHT OR HEAVY WATER (VERY IMPORTANT FUTURE).
  - ► HIGH RETENTION CAPABILITY FOR FISSION GASES FOR FUEL TEMPERATURES LESS THAN 1000 ° C.

- DISADVANTAGES:

  - ► LOW THERMAL CONDUCTIVITY, I.E., HIGH TEMPERATURE GRADIENT WITHIN FUEL: (10^6 $°C/cm$) .
  - ► HIGH CENTERLINE TEMPERATURE: 2000-2400 ° C.

TEMPERATURE DISTRIBUTION IN THE FUEL - FUEL MATERIALS-CERAMIC

- FABRICATION

  - CYLINDRICAL FUEL PELLETS ARE MANUFACTURED BY COMPACTING UNDER PRESSURE A FINELY GROUND URANIUM DIOXIDE POWDER.
  - PELLETS ARE THEN SINTERED IN A NEUTRAL ATMOSPHERE AT ABOUT 1700 ° C.
  - BY CONTROLLING THE SINTERING CONDITIONS, FUEL OF DENSITY BETWEEN 80% AND 98% OF THE THEORETICAL DENSITY CAN BE PRODUCED.
  - PELLETS ,AFTER GRINDING, ARE INSERTED IN A ZIRCALLOY TUBING AND SEALED BY WELDING THE END CAP.

- RESTRUCTURING

- DURING IRRADIATION, UNDER THE EFFECT OF HIGH TEMPERATURES AND TEMPERATURE GRADIENTS, UO_2 FUEL PELLETS UNDERGO CHANGES IN THE MORPHOLOGY.

- THESE HIGH TEMPERATURES AND GRADIENTS PRODUCE A RAPID DIFFUSIONS OF THE OXIDES TOWARD THE LOW TEMPERATURE REGIONS AND THE VOIDS (PORES) TOWARD THE HIGH TEMPERATURE CENTER REGION.

- THE RESULT IS THE RESTRUCTURING OF THE FUEL PELLET INTO CHARACTERISTIC ZONES:

REFER TO FIGURE 7.6

- ► **UNCHANGED REGION** WHERE THE TEMPERATURES ARE LOW AND INITIAL FINE GRAIN STRUCTURE IS KEPT.

- ► **EQUIAXED GRAINS REGION** WHERE THE INITIAL FINE GRAINS HAVE GROWN MANY TIMES OF THEIR ORIGINAL SIZE.

  IN THIS REGION THE TEMPERATURE IS THE SIGNIFICANT PARAMETER.

- ► **COLUMNAR GRAINS REGION** WHERE THE BOUNDARIES OF THE GRAINS ARE DELINEATED BY FINE RADIAL STREAKS OF MIGRATING PORES OR FISSION GAS BUBBLES.

  IN THIS REGION THE TEMPERATURE GRADIEN IS THE SIGNIFICANT PARAMETER.

- ► THE FORMATION OF A **CENTRAL VOID** IS ALSO POSSIBLE TO ACCOMMODATE THE MIGRATING VOIDS (PORES).

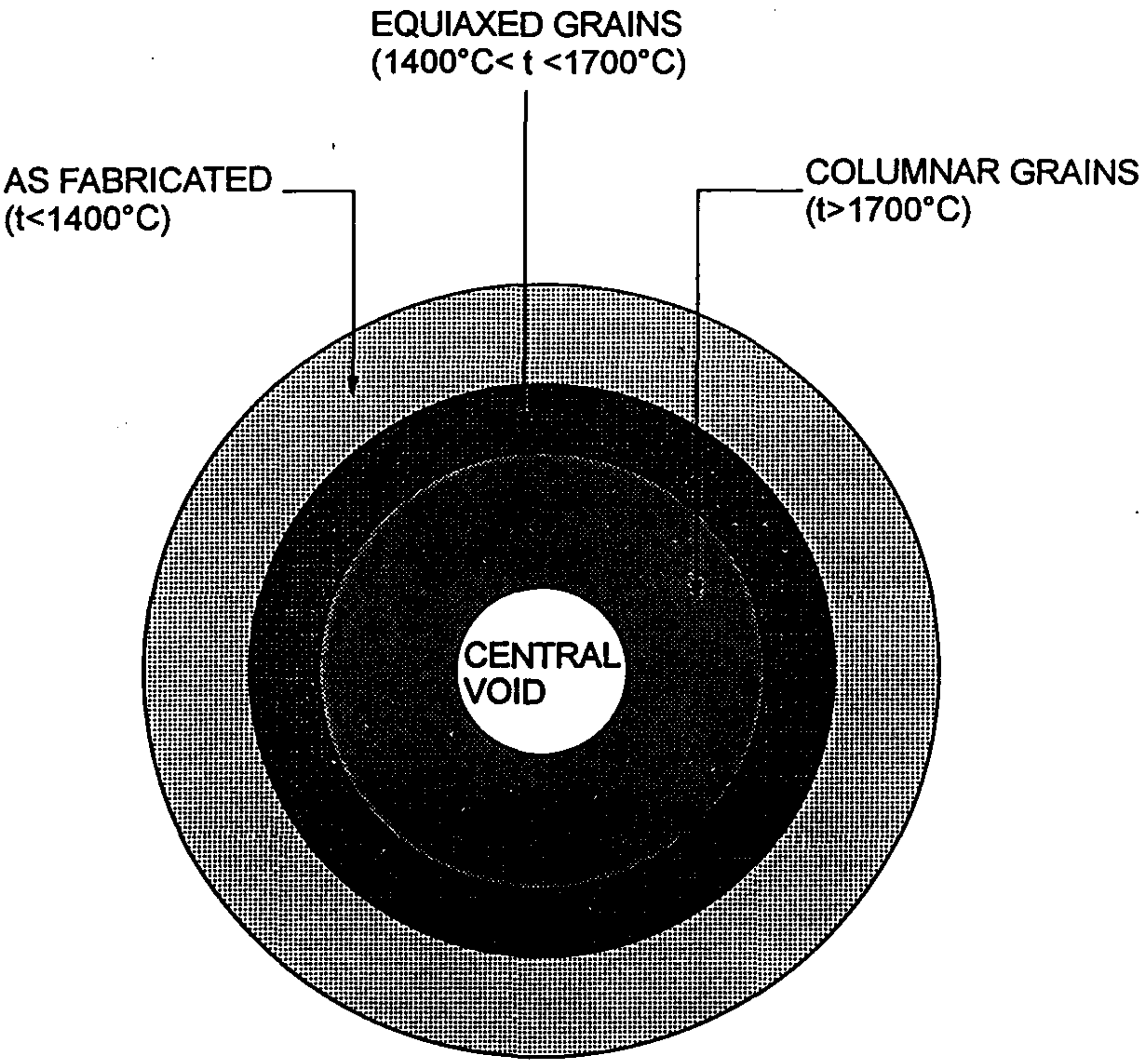


Figure 7.6 Restructuring of an oxide fuel pellet during the high temperature irradiation.

TEMPERATURE DISTRIBUTION IN THE FUEL - FUEL MATERIALS-CERAMIC

- UNDER THE NORMAL OPERATING CONDITIONS OF WATER COOLED REACTORS ONLY **UNCHANGED AND EQUIAXED REGIONS** ARE OBSERVED.

- CRACKING

- ► LARGE THERMAL GRADIENTS IN THE FUEL LEAD TO THERMAL STRESSES.
- ► THE CONSEQUENCE IS THE FORMATION OF RADIAL CRACKS.
- ► THESE CRACKS DONT AFFECT THE PERFORMANCE OF THE FUEL IF THE PELLETS ARE SUITABLY RESTRAINED BY THE CLADDING.

- SWELLING

- ► CONSTITUTE A MAJOR LIMITATION ON THE PERFORMANCE OF THE URANIUM DIOXIDE FUEL.
- ► THE SWELLING IS THE CONSEQUENCE OF THE FORMATION OF TWO FISSION PRODUCT ATOMS (MAINLY GASEOUS ONES) FOR EVERY FISSILE ATOM.
- ► AT LOW AND MODERATE BURNUPS SWELLING IS NOT SIGNIFICANT AND VARIES LINEARLY WITH BURNUP.
- ► ABOVE A CRITICAL BURNUP, SWELLING BECOMES IMPORTANT AND LEADS TO UNACCEPTABLE DIMENSIONAL CHANGES.
- ► CRITICAL BURNUP CAN BE INCREASED BY:
  1. DECREASING SLIGHTLY THE DENSITY OF THE URANIUM DIOXIDE, I.E., INCREASING THE POROSITY, AND
  2. PROVIDING MORE CLADDING RESTRAIN.
- ► **FIGURE 7.7** SHOWS THE EFFECT OF THE VOID (PORES) WITHIN THE FUEL ON THE CRITICAL BURNUP.
- ► CRITICAL BURNUP ALSO DEPENDS ON THE OPERATING TEMPERATURE OF THE FUEL.

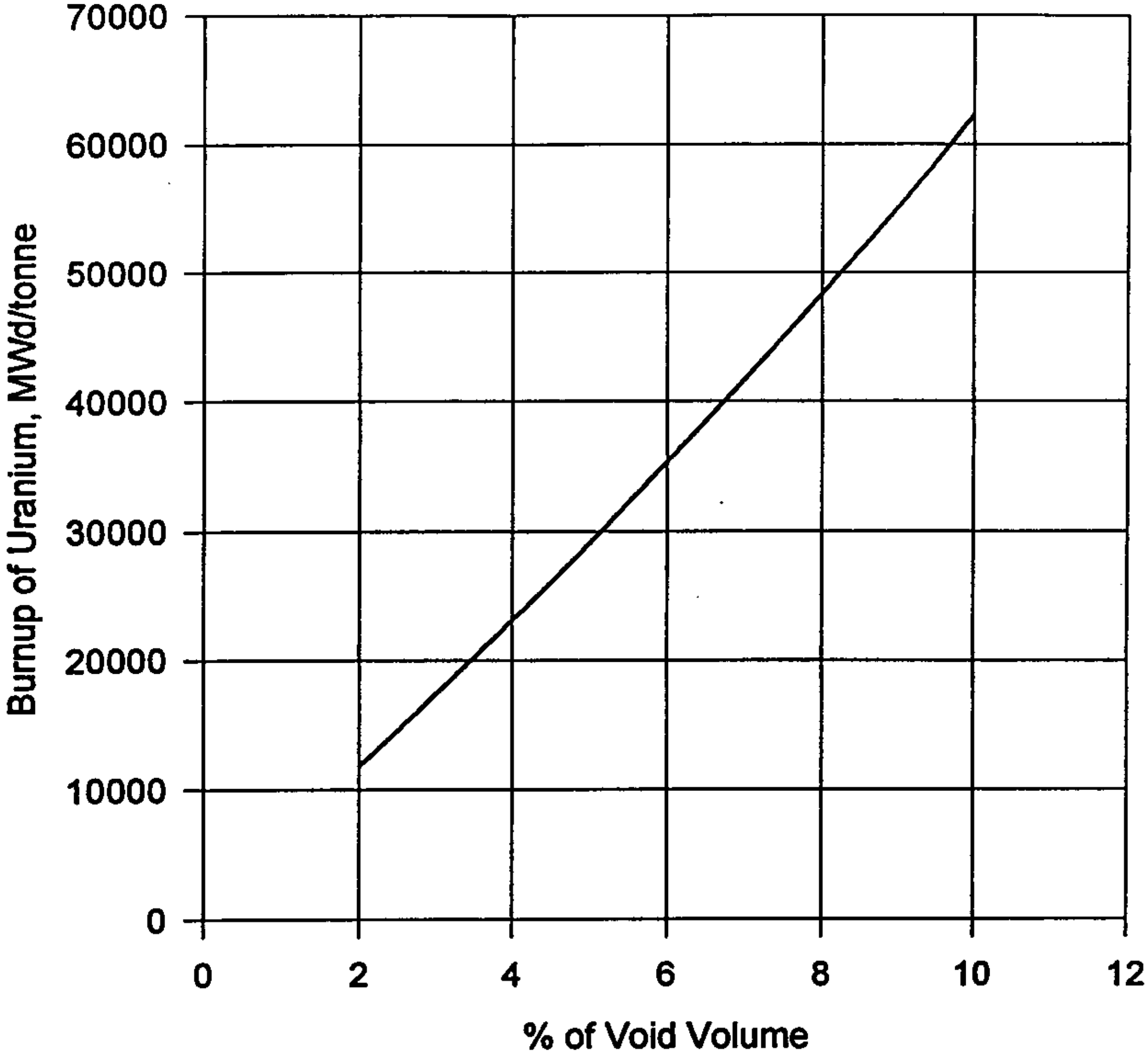


Figure 7.7 Effect of void within the fuel on the critical burnup.

THERMAL CONDUCTIVITY OF URANIUM DIOXIDE

- THE PROPERTIES OF URANIUM DIOXIDE IN GENERAL AND ITS CONDUCTIVITY IN PARTICULAR HAVE BEEN EXTENSIVELY STUDIED.

- HOWEVER, THE CONDUCTIVITY OF UO_2 IS NOT KNOWN ACCURATELY.

- THE CONDUCTIVITY OF CERAMIC FUEL DEPENDS ON:

  1. TEMPERATURE,
  2. POROSITY (DENSITY),
  3. U/O RATIO, AND
  4. FUEL BURNUP.

- THE RELATIVE DENSITY OF UO_2 IS AROUND 95%.

- THE RELATIVE DENSITY IS DEFINED AS THE RATIO OF THE TRUE DENSITY TO THE THEORETICAL DENSITY:

$$\rho_{\%} = \frac{\rho}{\rho_{TH}}$$

- THEORETICAL DENSITY, ρ_{TH} , IS THE DENSITY OF THE PORLESS SOLID.

- THE THEORETICAL DENSITY OF UO_2 IS 1097 kg/m^3.

TEMPERATURE DISTRIBUTION IN THE FUEL - FUEL MATERIALS-CERAMIC

(THERMAL CONDUCTIVITY OF URANIUM DIOXIDE)

- BY CONTROLLING THE SINTERING CONDITIONS, THE RELATIVE DENSITY CAN BE VARIED.
- THE CONDUCTIVITY OF POROUS MATERIALS DEPENDS ON THE RELATIVE DENSITY AND INCREASES WITH INCREASING RELATIVE DENSITY (OR DECREASING POROSITY).
- HENCE, HIGH RELATIVE DENSITY (LOW POROSITY) IS DESIRABLE TO MAXIMIZE THE CONDUCTIVITY.
- HOWEVER, LOW POROSITY INCREASES THE SWELLING DUE TO THE FISSION GASES AND DEFORMS THE FUEL.
- TO ACCOMMODATE THE FISSION GASES SOME POROSITY (ABOUT 5%) IS REQUIRED.
- THE POROSITY IS DEFINED AS:

$$\alpha = \frac{V_p}{V} = \frac{V - V_s}{V}$$

V : TOTAL VOLUME OF THE PELLET

V_p : VOLUME OF THE PORES

V_s : VOLUME OF THE SOLID

(THERMAL CONDUCTIVITY OF URANIUM DIOXIDE)

- DENSITY OF THE OXIDE FUEL:

$$\rho = \rho_s(1-\alpha) + \rho_g \alpha$$

KNOWING THAT:

$$\rho_g \alpha << \rho_s(1-\alpha)$$

$$\rho_s = \rho_{TD}$$

WE CAN ALSO WRITE FOR THE POROSITY:

$$\alpha = 1 - \frac{\rho}{\rho_{TD}} \quad \leftarrow \text{POROSITY}$$

ρ / ρ_{TD} IS THE RELATIVE DENSITY.

- THE EFFECT OF THE POROSITY ON THE CONDUCTIVITY IS GIVEN BY THE MAXWELL-EUCKEN FORMULA:

$$k = \frac{1-\alpha}{1+\beta\alpha} k_{TD}$$

k : CONDUCTIVITY OF THE POROUS FUEL.

k_{TP} : CONDUCTIVITY AT THEORETICAL DENSITY.

β : PORE SHAPE FACTOR; $\beta = 0.5$ FOR FUEL OF RELATIVE DENSITY 0.9 AND ABOVE; $\beta = 0.7$ FOR FUEL OF LOWER DENSITY.

(THERMAL CONDUCTIVITY OF URANIUM DIOXIDE)

- DATA ON THE THERMAL CONDUCTIVITY OF UNIRRADIATED URANIUM DIOXIDE IS A A FUNCTION OF TEMPERATURE IS SHOWN IN THE FIGURE BELOW.
- DATA FROM DIFFERENT SOURCES WERE CORRECTED TO 95% DENSITY.

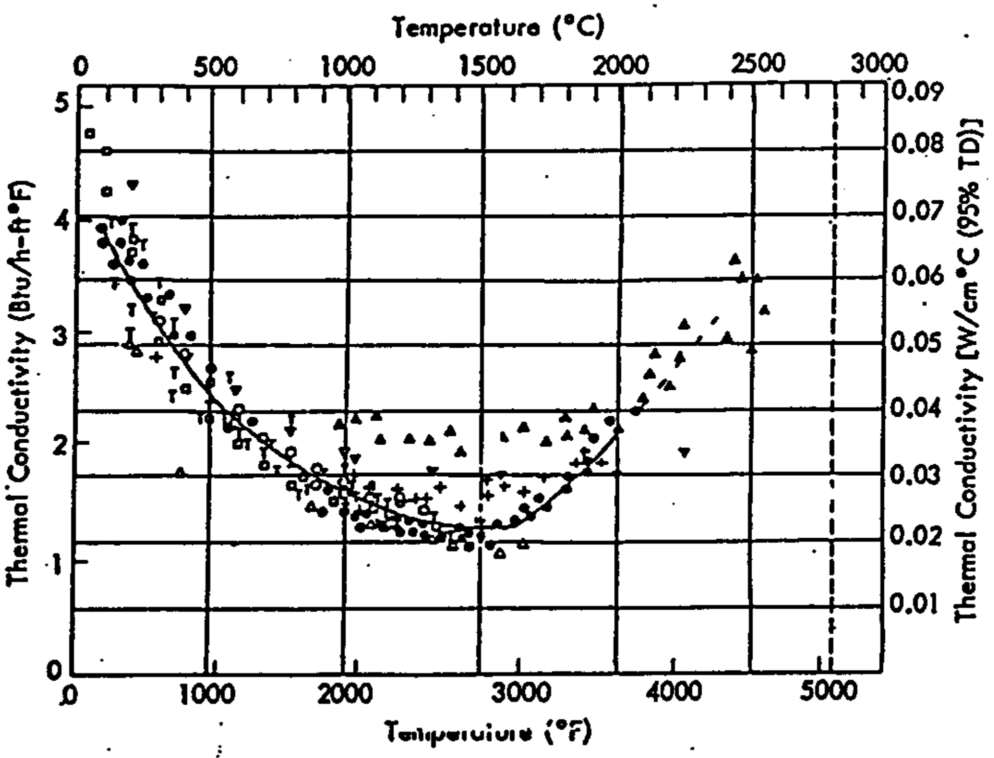


Fig. Thermal conductivity of unirradiated UO_2.

Notation ○ Godfrey et al., Ref. 27, 1964.

□ Dayton & Tipton, Battelle Memorial Institute Report BMI-1448, 1960.

▽ Kingery et al., *J. Am. Ceram. Soc.*, 37, 107, 1954.

T Howard & Galvin, UKAEA IG Report 51, 1960.

⊢ Reiswig, *J. Am. Ceram. Soc.*, 44, 48, 1961.

● Nishijima, Ref. 31, 1965.

+ Bush et al., *Trans. Am. Nucl. Soc.*, 7, 392, 1964.

▼ Feith, Ref. 32, 1963.

▲ Feith unpublished data.

(THERMAL CONDUCTIVITY OF URANIUM DIOXIDE)

- DESPITE THIS CORRECTION THE DATA IS STILL DISPERSED.
- THIS DISPERSION CAN BE SOMEWHAT REDUCED BY USING THE MAXWELL-EUCKEN FORMULA.
- AN OTHER CAUSE OF THE DISPERSION IS THE O/U RATIO WHICH VARIES BETWEEN 1.8 AND 2.2.
- THE CONDUCTIVITY OF URANIUM DIOXIDE INCREASES WHEN O/U IS REDUCED AND VICE VERSA.
- THE CONDUCTIVITY OF URANIUM DIOXIDE DECREASES WITH BURNUP.
- FOR AN UNIRRADIATED URANIUM DIOXIDE OF 95% THEORETICAL DENSITY THE CONDUCTIVITY IS GIVEN BY:

$$k_{95\%} = \frac{38.24}{402.4 + t} + 6.125 \times 10^{-13}(t + 273)^3 \quad W / cm\,^{\circ}C$$

t IS IN °C.

- TO APPLY THIS CORRELATION TO OTHER DENSITY RATIOS THE FOLLOWING CORRECTION SHOULD BE DONE:

$$\frac{k}{k_{95\%}} = \frac{1.025}{0.95}\frac{1-\alpha}{1-\alpha\beta}$$

TEMPERATURE DISTRIBUTION IN THE FUEL - FUEL MATERIALS-CERAMIC

(THERMAL CONDUCTIVITY OF URANIUM DIOXIDE)

- A CORRELATION WHICH TAKES INTO ACCOUNT THE BURNUP EFFECT AS WELL AS THE DENSITY EFFECT IS GIVEN BY:

$$k = \frac{(1-\alpha)}{(1+\alpha\beta)}\left(0.067 + 1.086\times 10^{-4}\,T + \frac{3.6}{T} + \frac{28.8}{T^2}\right.$$

$$\left. + 11.13\frac{F}{T}\right)^{-1} \quad W/m\,^{\circ}C$$

$$T = 1.8\,t + 491.7 \quad ; \; t \;\; in \;\; ^{\circ}C$$

$$F \;:\; burnup \;\; (fission/cm^3)\times 10^{-20}$$

- ONCE THE DEPENDENCE OF THE URANIUM DIOXIDE CONDUCTIVITY ON TEMPERATURE IS KNOWN THE FOLLOWING EQUATIONS:

$$\int_{t_s}^{t} k_f\,dt = \frac{q''' r_o^2}{4}\left[1 - \left(\frac{r}{r_o}\right)^2 - \left(\frac{r_i}{r_o}\right)^2 \ln\left(\frac{r_o}{r}\right)^2\right]$$

OR

$$\int_{t_s}^{t} k_f\,dt = \frac{q''' r_o^2}{4}\left[1 - \left(\frac{r}{r_o}\right)^2\right]$$

CAN BE EASILY INTEGRATED TO OBTAIN AN EQUATION WHOSE SOLUTION WILL YIELD THE TEMPERATURE PROFILE IN THE FUEL.

DETERMINATION OF THE CONDUCTIVITY OF URANIUM DIOXIDE

- IT IS DIFFICULT TO CONDUCT EXPERIMENTS TO DETERMINE THE CONDUCTIVITY OF URANIUM DIOXIDE.
- THE CONDUCTIVITY CAN BE DETERMINED BY INSERTING THERMOCOUPLES AT DIFFERENT RADIAL POSITIONS IN THE FUEL.
- THIS IS NOT AN EASY TASK.
- THE MEASUREMENTS ARE SUBJECT TO HIGH DEGREE OF UNCERTAINTY BECAUSE OF:
  - ► THE LOW THERMAL CONDUCTIVITY, AND
  - ► HIGH TEMPERATURE GRADIENTS.
- A MORE CONVENIENT WAY IS TO MEASURE:
  - ► THE CENTER TEMPERATURE, AND
  - ► FUEL SURFACE TEMPERATURE.
  - ► THE LATTER TEMPERATURE CAN ALSO BE CALCULATED FROM:
    - COOLANT TEMPERATURE,
    - LOCAL HEAT GENERATION RATE ,AND
    - ESTIMATED HEAT TRANSFER COEFFICIENT.
  - ► THE MEASUREMENT IS, THEREFORE, THE INTEGRAL OF CONDUCTIVITY.

TEMPERATURE DISTRIBUTION IN THE FUEL - FUEL MATERIALS-CERAMIC

(DETERMINATION OF THE CONDUCTIVITY OF URANIUM DIOXIDE)

- A THIRD METHOD ID BASED ON THE INDIRECT DETERMINATION OF THE OF THE FUEL CENTER TEMPERATURE BY TYING IT :
  - ► TO THE MELTING POINT (2880 °C), OR
  - ► TO THE OBSERVABLE CHANGE IN THE URANIUM DIOXIDE STRUCTURE SUCH AS GRAIN PATTERN MODIFICATION:

    - EQUIAXED (~1500 °C), OR
    - COLUMNAR (~1800 °).

- THE VALUE OF THE INTEGRAL OF CONDUCTIVITY FROM 0 °C TO MELTING POINT FOR A URANIUM DIOXIDE PELLET OF 95% IS:

$$\int_0^{2865} k\,dt = 93.5 \quad W/cm$$

TEMPERATURE DISTRIBUTION IN THE FUEL - FUEL MATERIALS-CERAMIC
CONDUCTIVITY OF URANIUM DIOXIDE

- THE DEPENDENCE OF THE INTEGRAL OF CONDUCTIVITY ON TEMPERATURE IS GIVEN IN:

FIGURE 7.9

- USING FIGURE 7.9 AND THE FOLLOWING EQUATION,

$$\int_0^{t_0} k\,dt - \int_0^{t_1} k\,dt = \frac{q'}{4\pi}$$

THE FUEL CENTER LINE TEMPERATURE IS DETERMINED AS FOLLOWS:

- ► FROM FIGURE 7.9 AND THE SURFACE TEMPERATURE OF THE FUEL PELLET DETERMINE THE INTEGRAL:

$$\int_0^{t_1} k\,dt$$

- ► SINCE q' IS KNOWN, DETERMINE FROM

$$\int_0^{t_0} k\,dt - \int_0^{t_1} k\,dt = \frac{q'}{4\pi}$$

THE INTEGRAL:

$$\int_0^{t_0} k\,dt$$

- ► USING FIGURE 7.9 DETERMINE THE CENTER LINE TEMPERATURE OF THE FUEL PELLET.

TEMPERATURE DISTRIBUTION IN THE FUEL - FUEL MATERIALS-CERAMIC

CONDUCTIVITY OF URANIUM DIOXIDE

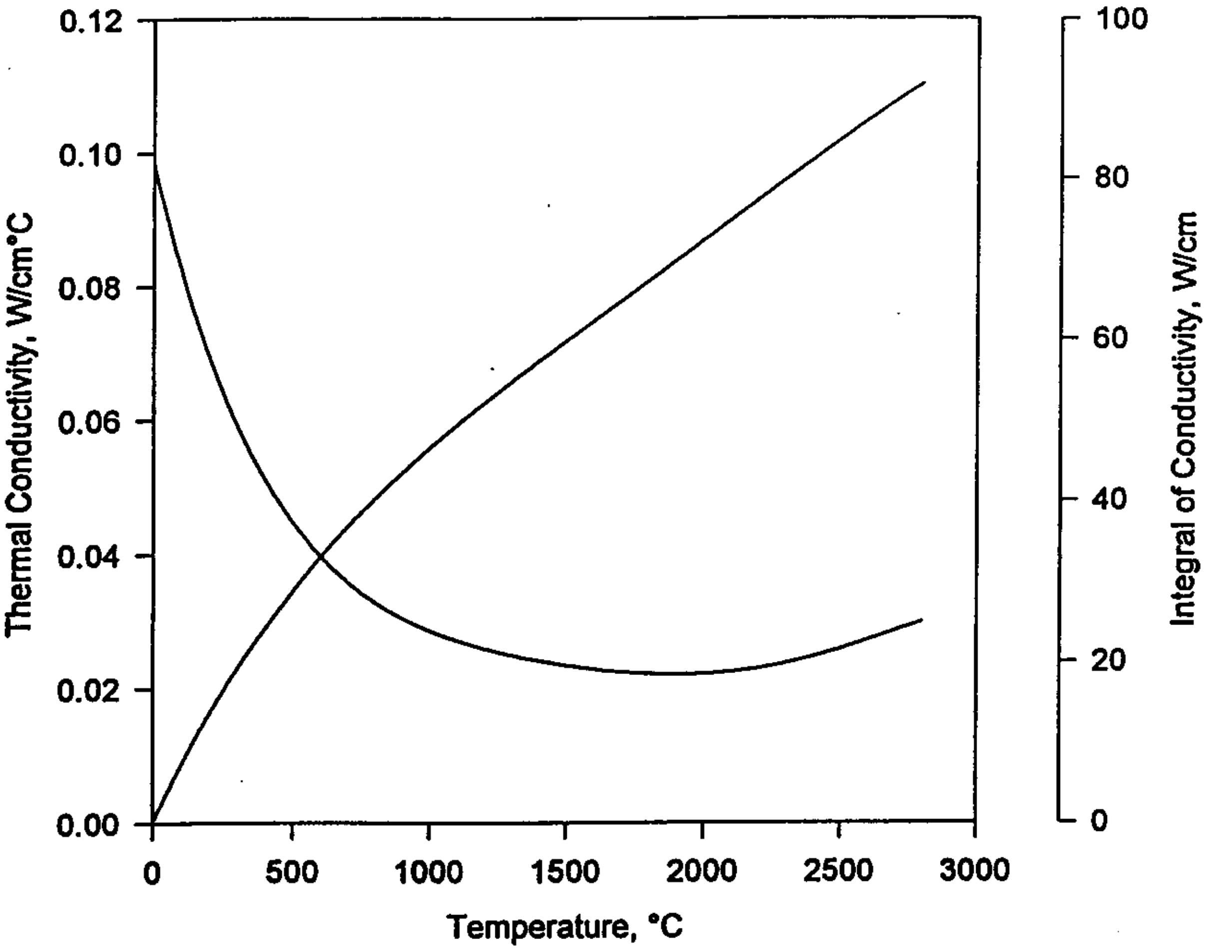


Figure 7.9 Conductivity integral of uranium dioxide pellets with 95% theoretical density.

TEMPERATURE DISTRIBUTION IN A FUEL ROD WITH NEUTRON DEPRESSION

- IN A FUEL ROD THE HEAT GENERATION RATE IS REDUCED WHEN PROGRESSED TOWARDS ITS CENTER.
- THIS IS DUE TO THE FACT THAT THE THERMALISATION OF THE FAST NEUTRONS TAKES PLACE IN THE MODERATOR.
- THERMAL NEUTRON ENTERING THE FUEL ARE FIRST ABSORBED BY THE OUTER PART OF THE FUEL.
- CONSEQUENTLY, THE NEUTRON FLUX IS LOWER IN THE CENTER.
- THE DISTRIBUTION OF THERMAL NEUTRONS WITHIN THE FUEL IS GIVEN BY:

$$\phi = \phi_o I_o(\kappa r)$$

I_o : MODIFIED BESSEL FUNCTION OF THE FIRS KIND AND ZERO ORDER. FIGURE ON PAGE 7,7

ϕ_o : thermal neutron flux on the center line of the fuel

I_o : zero order modified Bessel function of the first kind

κ : inverse of the thermal diffusion length in the fuel

- SINCE $q''' \sim \phi$

$$q''' = q_o''' I_o(\kappa r)$$

TEMPERATURE DISTRIBUTION IN THE FUEL WITH NEUTRON DEPRESSION

CONSIDER ONE DIMENSIONAL STEADY STATE HEAT CONDUCTION EQUATION:

$$\frac{1}{r}\left(\frac{d}{dr}k_f\, r\frac{dt}{dr}\right)+q'''=0$$

$$q'''=q_o'''I_o(\kappa r)$$

INTEGRATION
&
$$\int_o^r \kappa r\, I_o(\kappa r)\,dr = r\, I_1(\kappa r)$$

I_1: MODIFIED BESSEL FUNCTION OF THE FIRST KIND AND FIRST ORDER

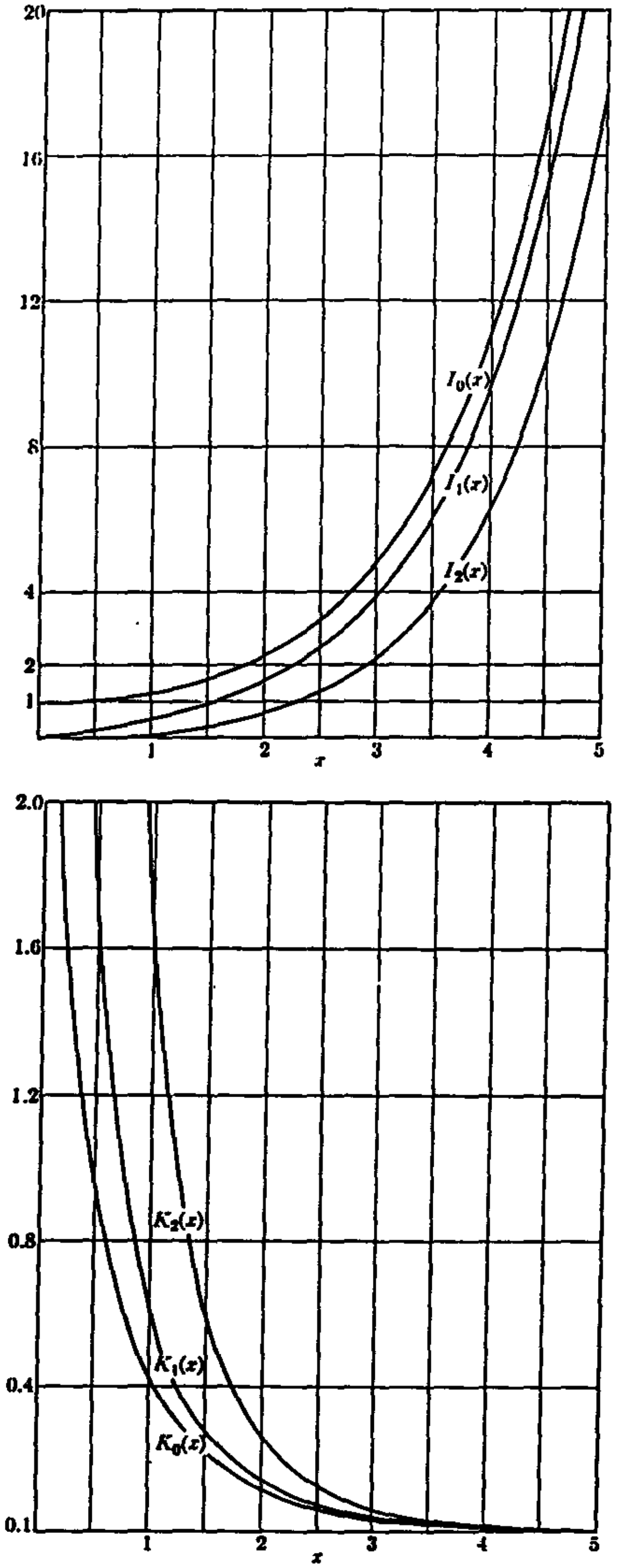
$$k r\frac{dt}{dr}=-q_o'''\int_o^r rI_o(\kappa r)\,dr=-\frac{q_o'''}{\kappa}r\, I_1(\kappa r)$$

SECOND INTEGRATION
&
$$\int \kappa\, I_1(\kappa r)\,dr = I_o(\kappa r)$$

$$\int_{t_s}^{t}k\,dt=-\frac{q_o'''}{\kappa^2}\int_{r_o}^{r}\kappa\, I_1(\kappa r)\,dr=\frac{q_o'''}{\kappa^2}\left[I_o(\kappa r_o)-I_o(\kappa r)\right]$$

$$r=0$$

$$\int_{t_s}^{t_o}k\,dt=\frac{q_o'''}{\kappa^2}\left[I_o(\kappa r_o)-1\right]$$



Modified Bessel functions of the first and second kind.

TEMPERATURE DISTRIBUTION IN THE FUEL WITH NEUTRON DEPRESSION

THE AVERAGE HEAT SOURCE (POWER DENSITY) OVER THE CROSS SECTION OF THE FUEL IS DEFINED AS:

$$\overline{q'''} = \frac{\int_0^{r_o} q''' 2\pi r\,dr}{\int_0^{r_o} 2\pi r\,dr} = \frac{2}{r_o^2}\int_0^{r_o} q''' r\,dr$$

$$q''' = q_o''' I_o(\kappa r)$$

$$\int_o^r \kappa r\, I_o(\kappa r)\,dr = r\, I_1(\kappa r)$$

$$\overline{q'''} = \frac{2 q_o''' I_1(\kappa r_o)}{\kappa r_o}$$

RECONSIDER:

$$\int_{t_f}^{t} k\,dt = \frac{q_o'''}{\kappa^2}\left[I_o(\kappa r_o) - I_o(\kappa r)\right]$$

$$\int_{t_f}^{t_o} k\,dt = \frac{q_o'''}{\kappa^2}\left[I_o(\kappa r_o) - 1\right]$$

$$\overline{q'''} = \frac{2 q_o''' I_1(\kappa r_o)}{\kappa r_o}$$

$$q' = \pi r_o^2 \overline{q'''}$$

TEMPERATURE DISTRIBUTION IN THE FUEL WITH NEUTRON DEPRESSION

$$\int_{t_s}^{t} k\,dt = \frac{q'}{4\pi}\frac{2}{\kappa r_o\, I_1(\kappa r_o)}\left[I_o(\kappa r_o) - I_o(\kappa r)\right]$$

$$\int_{t_s}^{t_o} k\,dt = \frac{q'}{4\pi}\left[\frac{2}{\kappa r_o}\frac{I_o(\kappa r_o)-1}{I_1(\kappa r_o)}\right]$$

CONSIDER AGAIN:

$$\int_{t_s}^{t_o} k\,dt = \frac{q'}{4\pi}\left[\frac{2}{\kappa r_o}\frac{I_o(\kappa r_o)-1}{I_1(\kappa r_o)}\right]$$

✓ OK

DEFINING

$$f = \left[\frac{2}{\kappa r_o}\frac{I_o(\kappa r_o)-1}{I_1(\kappa r_o)}\right]$$

$$\int_{t_s}^{t_o} k\,dt = f\,\frac{q'}{4\pi}$$

TEMPERATURE DISTRIBUTION IN THE FUEL WITH NEUTRON DEPRESSION

- f IS CALLED "**FLUX DEPRESSION FACTOR.**"
- f IS A FUNCTION OF:
  - THE FUEL ENRICHMENT, AND
  - THE FUEL ROD DIAMETER.
- FOR NATURAL URANIUM AND FOR LOW ENRICHED URANIUM (<2.5%), f IS SLIGHTLY LESS THAN 1.
- FOR HIGH ENRICHMENT f IS INCREASINGLY SMALLER THAN 1.
  - f VALUES FOR A URANIUM DIOXIDE PELLET OF 95% THEORETICAL DENSITY AND 9.4 mm IN ARE:

ENRICHMENT %	f
0.	1.
5	0.94
10	0.87
15	0.79
20	0.72

NATURAL URANIUM

$$f = \left[\frac{2}{\kappa r_o} \frac{I_o(\kappa r_o) - 1}{I_1(\kappa r_o)} \right]$$

$$\int_{t_s}^{t_o} k\,dt = f\,\frac{q'}{4\pi}$$

EFFECT OF f ON q' (LINEAR POWER DENSITY):

- INTEGRAL OF CONDUCTIVITY FOR UNIFORM HEAT GENERATION.	$\Rightarrow \int_{t_s}^{t_o} k\,dt = \left[\frac{q'}{4\pi}\right]_U$
- INTEGRAL OF CONDUCTIVITY FOR NON-UNIFORM HEAT GENERATION.	$\Rightarrow \int_{t_s}^{t_o} k\,dt = f\left[\frac{q'}{4\pi}\right]_{NU}$

- THE COMPARISON OF THE ABOVE INTEGRAL OF CONDUCTIVITY FOR THE SAME t_f *and* t_o YIELDS:

$$\frac{[q']_{NU}}{[q']_U} = \frac{1}{f}$$

- CONSEQUENTLY:

$$[q']_{NU} > [q']_U$$

REMARKS ON THE FUEL CHANNEL TEMPERATURE

- VARIATION OF THE TEMPERATURES ALONG THE CHANNEL:

FIGURE 7.10

- COMMENT ON THE COOLANT TEMPERATURE.
- COMMENTS ON THE TEMPERATURE OF:
    - THE OUTER SURFACE OF THE CLADDING,
    - THE FUEL SURFACE,
    - THE FUEL CENTERLINE.
- REASONS OF SUCH A VARIATION:
    - COOLANT TEMPERATURE INCREASES STEADILY PAST THE MID-PLANE OF THE CHANNEL.
    - LINEAR POWER (LINEAR HEAT FLUX) CHANGES VERY SLOWLY IN THE VICINITY OF THE CHANNEL MID-PLANE.
- AS WE MOVE AWAY FROM THE FUEL CENTER, THE MAXIMUM TEMPERATURES SHIFTS TOWARD THE EXIT OF THE FUEL CHANNEL.

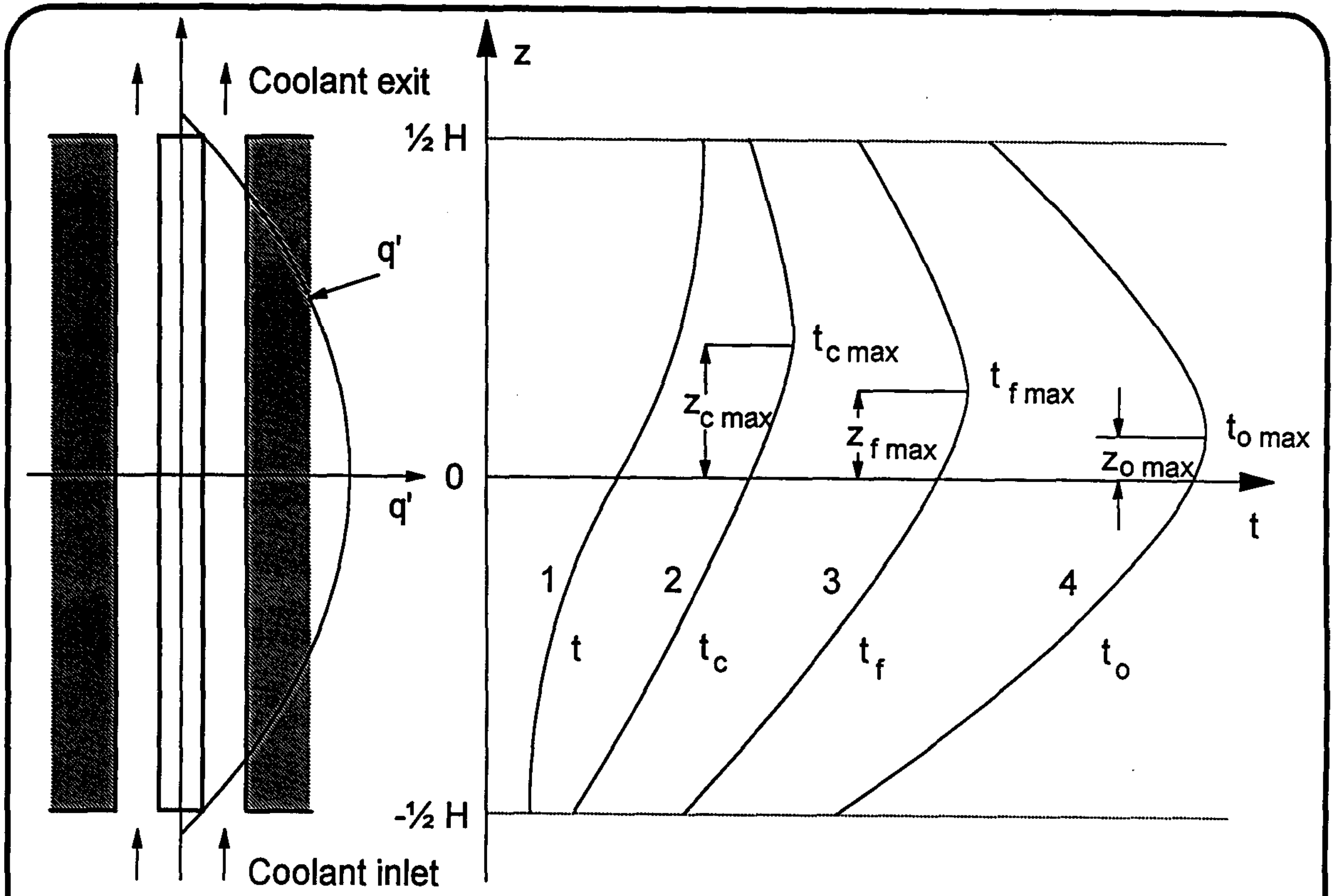


Figure 7.10 Axial temperature distributions in a fuel channel: 1) coolant, 2) cladding outer surface, 3) fuel surface and 4) fuel center-line

REMARKS ON CHANNEL TEMPERATURES

$$\frac{1}{\gamma''} = \frac{2\beta\,\bar{c}_p\dot{m}}{H}\left(\frac{1}{2\pi\, r_c\, h_c} + \frac{1}{2\pi\, k_c}\, ln\frac{r_c}{r_o} + \frac{1}{2\pi\, h_{con}} + \frac{1}{4\pi\, k_f}\right)$$

$$\longleftrightarrow$$

$$\boxed{\sim\frac{1}{\gamma}}$$

$$\longleftrightarrow$$

$$\boxed{\sim\frac{1}{\gamma'}}$$

$$\longleftrightarrow$$

$$\boxed{\sim\frac{1}{\gamma''}}$$

$$\Downarrow$$

$$\boxed{\begin{array}{l} \dfrac{1}{\gamma} < \dfrac{1}{\gamma'} < \dfrac{1}{\gamma''} \qquad or \\ \gamma > \gamma' > \gamma'' \end{array}}$$

$$\boxed{\begin{array}{l} z_m = a\, arctan\, b \\ b = \gamma \ \ ,or \ \ (=\gamma') \ \ ,or \ \ (=\gamma'') \end{array}}$$

CONSEQUENTLY:

$$\boxed{z_m \ \ increases\ as\ \ b \ \ increases}$$

REMARKS ON CHANNEL TEMPERATURES

- IN OTHER WORDS, THE LOCATION OF THE MAXIMUM TEMPERATURE IS RELATED TO THE THERMAL RESISTANCE.

- THIS LOCATION SHIFTS FURTHER DOWNSTREAM OF THE FUEL CHANNEL MID-PLANE AS THE THERMAL RESISTANCE DECREASES.

- OTHER COMMENTS:

  - CLADDING AND FUEL CENTERLINE TEMPERATURES TOGATHER WITH CHF IMPOSE LIMITATION ON THE MAXIMUM ALLOWABLE HEAT GENERATION IN THE FUEL ELEMENT.

  - THE POWER DENSITY PROFILE ALONG THE CHANNEL IS NOT ALWAYS SYMMETRICAL BECAUSE OF:

    - THE FUEL-MODERATOR DENSITY VARIATIONS,
    - THE FUEL LOADING AND UNLOADING (CANDU),
    - THE MOVEMENT OF THE CONTROL RODS.

  - PHYSICAL PROPERTIES OF THE COOLANT VARY WITH TEMPERATURE; CONSEQUENTLY h_c VARIES ALONG THE CHANNEL.

  - A CONSTANT h_c BASED ON THE AVERAGE CHANNEL TEMPERATURES YIELDS:

    - 5 TO 10 % ERROR ON h_c FOR LIQUID COOLANTS,
    - ~3 % ERROR ON h_c FOR GAS COOLANTS.

PRESSURE DROP IN FUEL CHANNELS AND PUMPING POWER

- PRESSURE DROP THROUGH THE FUEL CHANNEL
  - AS THE COOLANT FLOWS THOUGH THE CHANNEL ITS PRESSURE DROPS DUE TO:
    1. THE FRICTION BETWEEN THE COOLANT AND WALL,
    2. THE ACCELERATION AS THE SPECIFIC MASS OF THE COOLANT DECREASES WITH INCREASING TEMPERATURE, OR VAPOR FORMATION,
    3. THE GRAVITY.
  - ACCELERATION PRESSURE DROP IS SMALL IN LIQUID COOLED REACTORS BUT MAY BE SIGNIFICANT IN A GAS COOLED REACTOR OR IN A REACTOR WHERE BOILING TAKES PLACE.
- THE TOTAL PRESSURE DROP GRADIENT IN A FUEL CHANNEL IS GIVEN BY:

$$-\frac{dp}{dz}=\left(\frac{dp}{dz}\right)_f+\left(\frac{dp}{dz}\right)_a+\left(\frac{dp}{dz}\right)_g=\frac{4\tau_w}{D}+G\frac{d}{dz}\underset{\uparrow}{v}+\rho g$$

specifique volume

p : *pressure*, N/m^2

τ_w : *wall shear stress*, N/m^2

D : *diameter or hydraulic diameter of the channel*, m

G : *mass flux of the coolant*, $kg/m^2 s$

v : *specific volume of the coolant*, m^3/kg

ρ : *specific mass of the coolant*, kg/m^3

g : *acceleration of the gravity*, m/s^2

$$-\frac{dp}{dz}=\left(\frac{dp}{dz}\right)_f+\left(\frac{dp}{dz}\right)_a+\left(\frac{dp}{dz}\right)_g=\underbrace{\frac{4\tau_w}{D}}_{1}+\underbrace{G^2\frac{d}{dz}v}_{2}+\underbrace{\rho g}_{3}$$

- THE TERM 1 IS THE PRESSURE DROP DUE TO THE FRICTION EFFECT AND GIVEN BY:

$$\left(\frac{dp}{dz}\right)_f=\frac{4\tau_w}{D}=f\frac{1}{2}\rho U_m^2\frac{1}{D}=f\frac{1}{2}\frac{G^2}{\rho}\frac{1}{D}$$

f : is the friction factor.

$$\boxed{G=\rho U_m}$$

- THE TERM 2 IS THE ACCELERATION PRESSURE DROP.
- THE TERM 3 IS THE GRAVITATIONAL OR ELEVATION PRESSURE DROP.

- TOTAL PRESSURE DROP

$$\boxed{-\frac{dp}{dz}=\left(\frac{dp}{dz}\right)_f+\left(\frac{dp}{dz}\right)_a+\left(\frac{dp}{dz}\right)_g=\frac{4\tau_w}{D}+G\frac{d}{dz}v+\rho g}$$

INTEGRATION FROM $-\frac{1}{2}H$ to $\frac{1}{2}H$

CONSTANT PHYSICAL PROPERTIES ASSUMPTION: VISCOSITY AND SPECIFIC MASS

v_e

$$\Delta p_T = -\int_{-\frac{1}{2}H}^{\frac{1}{2}H} dp = \int_{-\frac{1}{2}H}^{\frac{1}{2}H} \frac{4\tau_w}{D}\,dz + \int_{-\frac{1}{2}H}^{\frac{1}{2}H} G^2 d(v) + \int_{-\frac{1}{2}H}^{\frac{1}{2}H} \rho\, g\,dz$$

v_i

- ACCELERATION PRESSURE DROP SMALL
- GRAVITATIONAL PRESSURE DROP SMALL

$$\Delta p_T \cong \frac{1}{2} f \frac{G^2}{\rho}\frac{H}{D}$$

- IF THE PHYSICAL PROPERTIES ARE NOT CONSTANT, THE TOTAL PRESSURE DROP CAN ONLY BE OBTAINED BY NUMERICAL INTEGRATION METHODS.

- FOR GAS COOLANTS, ACCELERATION TERM CAN NOT BE NEGLECTED.

$$\boxed{\Delta p_T = -\int_{-\frac{1}{2}H}^{\frac{1}{2}H} dp = \int_{-\frac{1}{2}H}^{\frac{1}{2}H} \frac{4\tau_w}{D} dz + \int_{-\frac{1}{2}H}^{\frac{1}{2}H} G^2 d(v) + \int_{-\frac{1}{2}H}^{\frac{1}{2}H} \rho g dz}$$

GRAVITY NEGLIGIBLE

$$\boxed{\left(\frac{dp}{dz}\right)_f = \frac{4\tau_w}{D} = \frac{1}{2} f \frac{G^2}{\rho} \frac{1}{D}}$$

$$\boxed{\Delta p_T = -\int_{-\frac{1}{2}H}^{\frac{1}{2}H} dp = \frac{1}{2} \frac{G^2}{D} \int_{-\frac{1}{2}H}^{\frac{1}{2}H} \frac{f}{\rho} dz + G^2 \int_{-\frac{1}{2}H}^{\frac{1}{2}H} d\left(\frac{1}{\rho}\right)}$$

- ► THE CHANGES IN PHYSICAL PROPERTIES OF THE GAS COOLANTS ARE DUE:
  - TO THE CHANGES IN TEMPERATURES
  - RATHER THAN THE CHANGES IN PRESSURES.
- ► FOR PRACTICAL PURPOSES, THE PRESSURE CAN BE TAKEN CONSTANT AND EQUAL TO:

$$\overline{p} = \frac{p_i + p_e}{2}$$

p_i : *inlet pressure*, p_e : *exit pressure.*

- THE VARIATION OF THE VISCOSITY OF GASES WITH THE TEMPERATURE IS SLOW.
- $f \sim Re^{-0.2} \sim \mu^{0.2} \qquad \left(Re = \frac{\rho U D}{\mu} \right)$
  - THE CHANGES IN f WITH TEMPERATURE IS EVEN SMALLER.
  - CONSEQUENTLY, f CAN BE TAKEN AS CONSTANT.
- SPECIFIC VOLUME OF THE GAS COOLANT

$$v = v(p,T)$$

TOTAL DERIVATIVE

$$dv = \left(\frac{\partial v}{\partial p} \right)_T dp + \left(\frac{\partial v}{\partial T} \right)_p dT$$

$$p = const = \overline{p} = \left(\frac{p_i + p_e}{2} \right)$$

$$dv = \frac{\partial v}{\partial T} dT$$

$$pv = RT$$

$$dv = \frac{R}{\overline{p}} dT$$

$\overline{p}\, v = RT$

$\frac{\partial v}{\partial T} = \frac{R}{\overline{p}}$

- ACCELERATION PRESSURE DROP

$$G^2\int_{-\frac{1}{2}H}^{\frac{1}{2}H} d\left(\frac{1}{\rho}\right) = G^2\int_{-\frac{1}{2}H}^{\frac{1}{2}H} dv$$

$$dv = \frac{R}{\overline{p}}\,dT$$

$$G^2\int_{T_i}^{T_e}\frac{R}{\overline{p}}\,dT = G^2\,\frac{R}{\overline{p}}\left(T_e - T_i\right)$$

$$\times\frac{\overline{T}}{\overline{T}}$$

$$\overline{T} = \frac{T_i + T_e}{2}$$

$$G^2\int_{T_i}^{T_e}\frac{R}{\overline{p}}\,dT = G^2\,\frac{R}{\overline{p}}\,\overline{T}\,\frac{T_e - T_i}{\overline{T}}$$

$$\overline{\rho} = \frac{\overline{p}}{R\overline{T}}$$

$$G^2\int_{-\frac{1}{2}H}^{\frac{1}{2}H} d\left(\frac{1}{\rho}\right) = \frac{G^2}{\overline{\rho}}\,\frac{T_e - T_i}{\overline{T}}$$

TOTAL PRESSURE DROP

$$\Delta p_T = -\int_{-\frac{1}{2}H}^{\frac{1}{2}H} dp = \frac{1}{2}\frac{G^2}{D}\int_{-\frac{1}{2}H}^{\frac{1}{2}H}\frac{f}{\rho}dz + G^2\int_{-\frac{1}{2}H}^{\frac{1}{2}H} d\left(\frac{1}{\rho}\right)$$

$$f = const.$$

$$\rho = \overline{\rho} = \frac{\overline{p}}{R\overline{T}}$$

$$G^2\int_{-\frac{1}{2}H}^{\frac{1}{2}H} d\left(\frac{1}{\rho}\right) = \frac{G^2}{\overline{\rho}}\frac{T_e - T_i}{\overline{T}}$$

$$\Delta p_T = p_i - p_e = \frac{1}{2}f\frac{G^2}{\overline{\rho}}\frac{H}{D} + \frac{G^2}{\overline{\rho}}\frac{(T_e - T_i)}{\overline{T}}$$

- A MORE ACCURATE SOLUTION REQUIRES TO TAKE INTO ACCOUNT THE EFFECT OF PRESSURE CHANGES ON THE SPECIFIC VOLUME OF THE GAS.

$$\boxed{-\frac{dp}{dz}=\frac{4\tau_w}{D}+G\frac{d}{dz}v+\rho g}$$

$$\boxed{\left(\frac{dp}{dz}\right)_f=\frac{4\tau_w}{D}=\frac{1}{2}f\frac{G^2}{\rho}\frac{1}{D}}$$

GRAVITY NEGLIGIBLE

$$\boxed{-dp=\frac{1}{\rho}G^2\left(\frac{1}{2}\frac{f}{D}dz+\frac{dv}{v}\right)}$$

$$\boxed{\frac{dv}{v}=\frac{dT}{T}-\frac{dp}{p}}$$

$$\boxed{-dp=\frac{1}{\rho}G^2\left(\frac{1}{2}\frac{f}{D}dz+\frac{dT}{T}-\frac{dp}{p}\right)}$$

THE INTEGRATION OF THIS EQUATION CAN ONLY BE CARRIED OUT NUMERICALLY.

- IN MOST OF THE GAS COOLED REACTORS THE TERM:

$$\frac{G^2}{\rho}\frac{dT}{T}$$

ACCOUNTS 10 TO 15% OF THE TOTAL PRESSURE DROP.

- THE TERM:

$$-\frac{G^2}{\rho}\frac{dp}{p}$$

ACCOUNTS FOR 1 TO 2% OF THE TOTAL PRESSURE DROP.

- IN THE PRELIMINARY DESIGN, $-\frac{G^2}{\rho}\frac{dp}{p}$ CAN BE NEGLECTED.

POWER REQUIRED TO CIRCULATE THE COOLANT - PUMPING POWER

- A PUMP IS USED TO CIRCULATE THE COOLANT THROUGH THE FUEL CHANNEL.
- THE PRESSURE RISE ACROSS THIS PUMP BALANCES THE DROP IN PRESSURE OF THE FLOWING COOLANT IN FUEL CHANNELS.
- TO PRODUCE THE PRESSURE RISE MECHANICAL ENERGY IS ADDED TO THE COOLANT IN THE PUMP.
- SUBSEQUENTLY, THIS MECHANICAL ENERGY IS TRANSFORMED INTO THERMAL ENERGY AND TOGATHER WITH THE THERMAL ENERGY FROM THE FUEL IS ADDED TO THE COOLANT.
- THE CONVERSION OF THE TOTAL THERMAL ENERGY INTO MECHANICAL (ELECTRICAL) ENERGY IS ACHIEVED WITH AN EFFICIENCY OF 30%.
- THE USE OF ELECTRICAL ENERGY TO DRIVE THE PUMPS AND SUBSEQUENT RECONVERSION INTO ELECTRICAL ENERGY WITH AN EFFICIENCY OF 30% REDUCES THE OVERALL EFFICIENCY OF THE POWER PLANT.
- WHEN POWER INPUT TO THE PUMPS INCREASES, THE SIZE AND THE COST ALSO INCREASE.

- IN LIQUID COOLED REACTORS, THE PUMPING POWER CORRESPONDS 3 TO 4% OF THE ELECTRICAL OUTPUT.
- IN GAS COOLED REACTORS, THE PUMPING POWER IS WITHIN 6 TO 15% OF THE ELECTRICAL OUTPUT.
- THE PUMPING POWER IS CALCULATED BY USING THE ENERGY CONSERVATION PRINCIPLE FOR A FLOWING SYSTEM:

$$\boxed{Q + W = \dot{m}(h_2 - h_1) + \frac{1}{2}\dot{m}\left(w_2^2 - w_1^2\right) + \dot{m}g(z_2 - z_1)}$$

KINETIC ENERGY NEGLIGIBLE

POTENTIAL ENERGY NEGLIGIBLE

NO HEAT EXCHANGE WITH WORKING FLUID IN THE PUMP, I.E., Q= 0.

$$\boxed{W = \dot{m}(h_2 - h_1)}$$

$$W = \dot{m}(h_2 - h_1)$$

REMEMBERING THE THERMODYNAMIC RELATIONSHIP (THE SECOND *TdS* EQUATION):

$$dh = Tds + vdp$$

ISENTROPIC PROCESS:

$$dh \cong vdp$$

INTEGRATION BETWEEN INLET AND EXIT OF THE PUMP ASSUMING CONSTANT SPECIFIC VOLUME.

$$\int_{h_1}^{h_2} dh = \int_{p_1}^{p_2} v\, dp$$

OR

$$h_2 - h_1 = v(p_2 - p_1) = v\,\Delta p$$

THIS IS THE PUMPING POWER →

$$W = \dot{m} v\,\Delta p$$

$$\boxed{W = \dot{m} v \, \Delta p}$$

- THE ABOVE EQUATION GIVES THE POWER NECESSARY TO CIRCULATE THE COOLANT.
- THE POWER REQUIRED TO DRIVE THE PUMP IS GREATER THAN THIS BY A FACTOR:

$$\frac{1}{\eta_p}$$

- η_p IS THE COMBINED MECHANICAL AND ELECTRICAL EFFICIENCY OF THE PUMP MOTOR.
- THE PUMPING POWER IS THEN GIVEN BY:

$$\mathbf{P}_p = \frac{W}{\eta_p} = \frac{\dot{m} v \, \Delta p}{\eta_p}$$

  - ► TO MINIMIZE THE PUMPING POWER, THE SPECIFIC VOLUME SHOULD BE AS SMALL AS POSSIBLE.
  - ► CONSEQUENTLY, THE CIRCULATING PUMPS SHOULD BE PLACED TO THE COOLEST POINT OF THE CIRCUIT, I.E., INLET OF THE REACTOR.
  - ► IN THE PRELIMINARY DESIGN, TO ACCOUNT FOR THE PRESSURE DROP IN THE CIRCUIT OUTSIDE THE FUEL, THE PUMPING POWER MAY MULTIPLIED BY A FACTOR **e** (>1).

NET MECHANICAL POWER OF THE FUEL CHANNEL

- HEAT ENERGY PRODUCED IN THE CHANNEL (REACTOR CORE) IS USED TO PRODUCE ELECTRICITY.
- DURING THIS CONVERSION, THE TEMPERATURE OF THE COOLANT DECREASES:

  $$\text{from } t_e \text{ to } t_i$$

  AND HEAT IS EXTRACTED FROM THE COOLANT
- ACCORDING TO SECOND LAW OF THERMODYNAMICS, IT IS IMPOSSIBLE TO CONVERT THE EXTRACTED HEAT ENTIRELY AND CONTINUOUSLY INTO THE WORK.
- THEREFORE, WE ENDEAVOR TO CONVERT AS MUCH AS POSSIBLE THIS HEAT INTO MECHANICAL ENERGY.
- THE THERMODYNAMIC EFFICIENCY OF A POWER CYCLE IS:

  $$\frac{\text{work output}}{\text{heat received}}$$

- THE BASIC THERMODYNAMIC CYCLE IS THE **CARNOT** CYCLE. IN THIS CYCLE:
  - ALL HEAT IS RECEIVED ISOTHERMALLY AT A TEMPERATURE, T_h .
  - ALL HEAT IS REJECTED ISOTHERMALLY AT A LOWER TEMPERA TURE, T_c .
  - ALL THE PROCESSES IN THE CYCLE ARE REVERSIBLE.
  - THERMAL EFFICIENCY OF THE **CARNOT** CYCLE IS GIVEN BY:

[illegible]
the
[illegible]
[illegible]
[illegible]

$$\eta_c = 1 - \frac{T_c}{T_h}$$

- WHEN THE TEMPERATURE OF THE COOLANT LEAVING THE REACTOR DECREASES

$$from \quad t \quad to \quad t - dt$$

IT RELEASES PER UNIT OF MASS A HEAT ENERGY EQUAL TO THE

$$the\ change\ of\ enthalpy, \quad dh.$$

- THIS HEAT ENERGY CAN BE CONVERTED TO WORK WITH A MAXIMUM EFFICIENCY GIVEN BY:

$$\eta_C = 1 - \frac{T_c}{T}$$

- TOTAL HEAT RELEASED BY THE COOLANT WHEN ITS TEMPERATURE DECREASES

$$from\ t_e\ to\ t_i\ \ (or\ enthalpy\ from\ h_e\ to\ h_i)$$

IS GIVEN BY:

$$\mathbf{P}_c = \int_{h_i}^{h_e} dh = \int_{T_i}^{T_e} \bar{c}_p \, dT$$

WHERE T IS THE ABSOLUTE TEMPERATURE.

- THE MAXIMUM WORK THAT CAN BE RECOVERED FROM THIS HEAT IS:

$$W_{\max} = \int_{T_i}^{T_e} \left(1 - \frac{T_c}{T}\right) dh = \int_{T_i}^{T_e} \left(1 - \frac{T_c}{T}\right) \bar{c}_p \, dT$$

- THE EFFICIENCY OF THIS **IDEAL CONVERSION** IS GIVEN BY

$$\eta_c = \frac{W_{max}}{P_c} = \frac{\int_{T_i}^{T_e} \left(1 - \frac{T_c}{T}\right) \bar{c}_p \, dT}{\int_{T_i}^{T_e} \bar{c}_p \, dT} = 1 - \frac{T_c}{T_{av}}$$

- T_{av} IS THE **LOGARITHMIC MEAN TEMPERATURE** OF THE COOLANT DEFINED AS:

$$T_{av} = \frac{T_e - T_i}{\ln \frac{T_e}{T_i}}$$

$$T_{av} = \frac{T_e - T_i}{\ln \frac{T_e}{T_i}}$$

IF THE RATIO

$$\frac{T_e}{T_i}$$

IS CLOSE TO 1, AND USING THE APPROXIMATION:

$$\ln x \cong 2\frac{x-1}{x+1}$$

WHERE:

$$x = \frac{T_e}{T_i}$$

$$T_{av} = T_m = \frac{T_e + T_i}{2}$$

- THE EFFICIENCY OF THE IDEAL CONVERSION IS THEN GIVEN BY:

$$\eta_c = 1 - \frac{T_c}{T_m}$$

- ► IN CANDU REACTORS, TYPICAL INLET AND EXIT TEMPERATURES OF THE COOLANT ARE 267 °C AND 312 °C, RESPECTIVELY.
- ► THE CORRESPONDING **LOGARITHMIC MEAN TEMPERATURE** IS: 289.2 ° C.
- ► THE **ARITHMETIC MEAN TEMPERATURE** IS: 289.5 °C.
- ► THEREFORE T_{av} CAN BE REPLACED BY T_m .

- WE HAVE SEEN THAT FOR A IDEAL ENERGY CONVERSION SYSTEM, THE HEAT ENERGY CONVERTED INTO MECHANICAL ENERGY IS:

$$\mathbf{W}_{max} = \eta_c \, \mathbf{P}_c$$

- HOWEVER, REAL ENERGY CONVERSION SYSTEMS ARE NOT PERFECT AND HAVE SEVERAL IRREVERSIBILITIES SUCH AS:
  - ► FRICTION,
  - ► HEAT TRANSFER WITH FINITE TEMPERATURE DIFFERENCE,
  - ► ETC.

HEAT REMOVAL FROM REACTORS - CHANNEL NET MECHANICAL POWER

- THEREFORE, THE EFFICIENCY OF THE REAL SYSTEMS IS LESS THAN THE EFFICIENCY OF THE IDEAL SYSTEMS.

- INTRODUCING A FACTOR " $\eta_r < 1$ " TO TAKE INTO ACCOUNT THE IRREVERSIBILITIES, THE **RECOVERABLE MECHANICAL ENERGY** IS GIVEN BY:

$$W = \eta_r \eta_c P_c = \eta P_c$$

- η IS THE COMBINED THERMAL EFFICIENCY OF
  - THE STEAM CYCLE, AND
  - THE TURBO GENERATORS.

- THE EFFICIENCY OF NUCLEAR POWER PLANTS IS ABOUT 30%.

- P_c SHOWS THE **THERMAL OUTPUT** OF THE FUEL CHANNEL.

- IN CALCULATING THE **TOTAL THERMAL OUTPUT** OF THE CHANNEL, P_T, **THE POWER TO CIRCULATE THE COOLANT,** $\eta_p P_p$, SHOULD BE ADDED TO **THE THERMAL POWER OF THE CHANNEL**:

$$P_T = P_c + \eta_p P_p$$

- THE **GROSS ELECTRICAL POWER** IS THEN GIVEN BY:

$$\eta\left(P_c + \eta_p P_p\right)$$

- THE **NET ELECTRIC POWER** IS:

$$\mathbf{W}_{net} = \eta\left(\mathbf{P}_c + \eta_p \mathbf{P}_p\right) - \mathbf{P}_p = \eta \mathbf{P}_c - \left(1 - \eta\eta_p\right)\mathbf{P}_p$$

- THE OVERALL EFFICIENCY OF THE FUEL CHANNEL IS:

$$\eta_{net} = \frac{\mathbf{W}_{net}}{\mathbf{P}_c} = \eta - \left(1 - \eta\,\eta_p\right)\frac{\mathbf{P}_p}{\mathbf{P}_c}$$

  - THE EFFICIENCY OF THE PUMPS VARIES BETWEEN: 65% AND 95%.

  - IN **PWR AND CANDU** REACTORS THE EFFICIENCY OF THE PUMPS IS BETWEEN 90% AND 92%.

OPTIMUM OPERATING POINT OF A CHANNEL

- TO DISCUSS THE OPTIMUM OPERATING POINT OF THE CHANNEL IT WILL BE ASSUMED THAT:

$$\eta_p \mathbf{P}_p < \mathbf{P}_c$$

- THEREFORE WE WRITE:

$$\mathbf{P}_T \cong \mathbf{P}_c$$

- THE NET ELECTRICAL POWER OF THE CHANNEL BECOMES:

$$\boxed{\mathbf{W}_{net} \cong \eta \mathbf{P}_c - \mathbf{P}_p}$$

HEAT REMOVAL FROM REACTORS - CHANNEL NET MECHANICAL POWER

$$\mathbf{W}_{net} \cong \eta \mathbf{P}_c - \mathbf{P}_p$$

$$\boxed{\mathbf{W}_{net} \cong \eta \mathbf{P}_c - \mathbf{P}_p}$$

$$\boxed{\mathbf{P}_p = \frac{\dot{m} v \, \Delta p}{\eta_p}}$$

$$\boxed{\Delta p \cong \frac{1}{2} f \frac{G^2}{\rho} \frac{H}{D}}$$

$$\boxed{G = \frac{\mathbf{P}_c}{\bar{c}_p (t_e - t_i) A}}$$

$$\boxed{\Delta p_T = \frac{1}{2} f \frac{\mathbf{P}_c^2}{(t_e - t_i)^2} \frac{1}{\rho \bar{c}_p^2} \frac{H}{D A^2}}$$

$$\boxed{f = 0.184 \, \mathrm{Re}^{-0.2}}$$

$$\boxed{Re = \frac{GD}{\mu}}$$

$$\boxed{\Delta p = 0.092 \frac{\mathbf{P}_c^{1.8}}{(t_e - t_i)^{1.8}} \frac{\mu^{0.2}}{\rho \bar{c}_p^{1.8}} \frac{H}{D^{1.2} A^{1.8}}}$$

$$\mathbf{W}_{net} \cong \eta \mathbf{P}_c - \mathbf{P}_p$$

$$\mathbf{P}_p = \frac{\dot{m} v \, \Delta p}{\eta_p}$$

$$\Delta p = 0.092 \frac{\mathbf{P}_c^{1.8}}{(t_e - t_i)^{1.8}} \frac{\mu^{0.2}}{\rho \bar{c}_p^{1.8}} \frac{H}{D^{1.2} A^{1.8}}$$

$$\mathbf{P}_p = 0.092 \frac{1}{\eta_p} \frac{\mathbf{P}_c^{2.8}}{(t_e - t_i)^{2.8}} \frac{\mu^{0.2}}{\rho^2 \bar{c}_p^{2.8}} \frac{H}{A^{1.8} D^{1.2}}$$

$$\mathbf{W}_{net} = \eta \mathbf{P}_c - 0.092 \frac{1}{\eta_p} \frac{\mathbf{P}_c^{2.8}}{(t_e - t_i)^{2.8}} \frac{\mu^{0.2}}{\rho^2 \bar{c}_p^{2.8}} \frac{H}{A^{1.8} D^{1.2}}$$

$$a = 0.092 \frac{1}{\eta_p} \frac{1}{(t_e - t_i)^{2.8}} \frac{\mu^{0.2}}{\rho^2 \bar{c}_p^{2.8}} \frac{H}{A^{1.8} D^{1.2}}$$

$$\mathbf{W}_{net} = \eta \mathbf{P}_c - a \mathbf{P}_c^{2.8}$$

$$\mathbf{W}_{net} = \eta \mathbf{P}_c - a\mathbf{P}_c^{2.8}$$

IF THE AVERAGE COOLANT TEMPERATURE IS KEPT CONSTANT BY INCREASING THE COOLANT MASS FLOW RATE WITH THE CHANNEL POWER, "a" CAN BE TAKEN AS CONSTANT.

MAXIMUM NET ELECTRICAL OUTPUT OF THE CHANNEL IS OBTAINED BY WRITING:

$$\frac{d\mathbf{W}_{net}}{d\mathbf{P}_c} = 0 \Rightarrow \eta - 2.8\, a\, \mathbf{P}_c^{1.8} = 0$$

$$or \quad \mathbf{P}_c = \frac{\eta^{0.56}}{1.77a^{0.56}}$$

MAXIMUM NET ELECTRICAL OUTPUT OF THE CHANNEL

- BEYOND THE MAXIMUM THERMAL CHANNEL POWER:

$$P_c = \frac{\eta^{0.56}}{1.77a^{0.56}}$$

THE **NET ELECTRICAL POWER** FROM THE CHANNEL STARTS FALLING BECAUSE OF THE HIGH POWER REQUIRED TO CIRCULATE THE COOLANT.

THE CHOICE OF COOLANT FOR REACTORS

- THE COOLANT MAY BE A GAS (CARBON DIOXIDE OR HELIUM) OR LIQUID (LIGHT OR HEAVY WATER OR AN EUTECTIC SODIUM-POTAS~~S~~SIUM ALLOY).

- COOLANT SELECTION CRITERIA:

  - **ECONOMICS:**

    LOW COST, AVAILABILITY, LOW PUMPING POWER, HIGH HEAT TRANSFER COEFFICIENT.

  - **PHYSICS:**

    LOW FUSION POINT, HIGH BOILING TEMPERATURE, COMPATIBILITY WITH FUEL, CLADDING, VALVES, PIPING, ETC., GOOD THERMAL STABILITY.

  - **NEUTRONICS:**

    LOW NEUTRON CAPTURE CROSS-SECTION, MODERATING POWER COMPATIBLE WITH THE TYPE OF THE REACTOR (THERMAL OR FAST), LOW RADIOACTIVITY, GOOD STABILITY UNDER IR RADIATION.

- HELIUM AND CARBON DIOXIDE ARE VERY SUITABLE GASES

  - THE COST OF HELIUM IS HIGH.

- LIGHT AND HEAVY WATER ARE SUITABLE FOR THERMAL REACTORS:
  - ► PLAY DUAL ROLE: MODERATING AND COOLING.
  - ► HIGH TEMPERATURES IN BOTH FLUIDS REQUIRE HIGH SYSTEM PRESSURES:
    - THICK PRESSURE VESSELS OR TUBES.
  - ► THE CAPTURE CROSS-SECTION OF LIGHT WATER IS HIGH:
    - ENRICHED URANIUM IS REQUIRED TO ACHIEVE THE CRITICALITY.
  - ► HEAVY WATER HAS VERY LOW CAPTURE CROSS-SECTION BUT IT IS EXPENSIVE.
  - ► BOTH WATERS REQUIRE STAINLESS STEEL OR ZIRCALLOY CLADDING.
- LIQUID METALS - SODIUM- POTASSIUM ALLOY
  - ► ADVANTAGES:
    - HAVE HIGH SATURATION TEMPERATURE AT ATMOSPHERIC PRESSURE - A PRESSURIZED COOLANT SYSTEM IS NOT REQUIRED.
    - DO NOT MODERATE FAST NEUTRONS; SUITABLE FOR FAST REACTORS.
  - ► DISADVANTAGES:
    - RADIATION HAZARD BECAUSE OF $^{23}Na(n,\gamma)^{24}Na$ REACTION.
    - VIOLENT CHEMICAL REACTION WITH AIR AND WATER.
    - HIGH NEUTRON CAPTURE CROSS-SECTION FOR THERMAL NEUTRONS - NOT SUITABLE FOR THERMAL REACTORS.

- A GOOD COMPARISON OF THE PERFORMANCE OF DIFFERENT COOLANTS CAN BE UNDERTAKEN BY EXAMINING THE RATIO OF:

$$\frac{\textit{pumping power}}{\textit{channel thermal power}}$$

WHICH HAS THE FOLLOWING FORM:

$$\frac{\mathbf{P}_p}{\mathbf{P}_c} = \underbrace{\left[0.092\frac{1}{\eta_p}\frac{\mathbf{P}_c^{1.8}}{(t_e - t_i)^{2.8}}\right]}_{1}\underbrace{\left[\frac{\mu^{0.2}}{\rho^2\,\bar{c}_p^{2.8}}\right]}_{2}\underbrace{\left[\frac{H}{D^{1.2}\,A^{1.8}}\right]}_{3}$$

  - THE TERM 1 IS RELATED TO THE OPERATING POINT OF THE CHANNEL.
  - THE TERM 2 IS RELATED TO THE PHYSICAL PROPERTIES OF THE COOLANT.
  - THE TERM 3 IS RELATED TO THE GEOMETRY OF THE CHANNEL.

- FROM COOLANT POINT OF VIEW, THE TERM 2 SHOULD BE AS SMALL AS POSSIBLE TO ENSURE A MINIMUM:

$$\frac{\mathbf{P}_p}{\mathbf{P}_c}$$

- A BETTER DISCUSSION OF THE COOLANT EFFECT CAN BE CARRIED OUT BY WRITING THAT:

$$\mathbf{P}_c = h_c\,S\,\Delta t_f$$

WHERE

h_c : heat transfer coefficient between the rod and coolant,
S : heat transfer surface,
Δt_f : average temperature difference between the fuel element surface and coolant.

ASSUMING THAT HEAT TRANSFER COEFFICIENT IS GIVEN BY:

$$h_c = 0.023\frac{k_f}{D}\left(\frac{GD}{\mu}\right)^{0.8}\left(\frac{\bar{c}_p\mu}{k_f}\right)^{0.4}$$

THE RATIO:

$$\frac{\mathbf{P}_p}{\mathbf{P}_c}$$

CAN BE WRITTEN AS:

$$\frac{\mathbf{P}_p}{\mathbf{P}_c} = 4\underbrace{\left[\frac{1}{DSA}\right]}_{1}\underbrace{\left[\frac{\mathbf{P}_c^2}{(\Delta t)^2\Delta t_f}\right]}_{2}\underbrace{\left[\frac{1}{\rho^2\bar{c}_p^2}\left(\frac{k}{\bar{c}_p\mu}\right)^{0.4}\frac{\mu}{k_f}\right]}_{3}$$

WHERE $\Delta t = (t_e - t_i)$.

- IN THE ABOVE EQUATION, THE PHYSICAL PROPERTIES OF THE COOLANT APPEAR IN THE TERM 3.

- IN ORDER TO ACHIEVE MINIMUM RATIO OF:

$$\frac{\mathbf{P}_p}{\mathbf{P}_c}$$

THE TERM 3 SHOULD BE AS SMALL AS POSSIBLE.

- TABLE 7.7 COMPARES SEVERAL COOLANTS FROM THE POINT OF VIEW OF PHYSICAL PROPERTIES.
- THE PROPERTIES WERE CALCULATED AT:
  - 0 °C FOR GASES, AND
  - 80 °C FOR LIQUIDS
- IT IS OBSERVED THAT HYDROGEN IS THE BEST GAS COOLANT FOLLOWED BY HELIUM AND STEAM. HOWEVER:
  - HYDROGEN IS EXPLOSIVE,
  - HELIUM IS EXPENSIVE,
  - STEAM IS CORROSIVE.
- CARBON DIOXIDE IS A SUITABLE COOLANT FOR GAS COOLED REACTORS.
- AIR AND NITROGEN HAVE ACTIVATION PROBLEMS.
- AMONG LIQUIDS WATER (LIGHT AND HEAVY) IS THE BEST, FOLLOWED BY LIQUID METALS.

Table 7.7 Comparison of the coolants

Gas	$\left[\frac{1}{\rho^2 \bar{c}_p^2}\left(\frac{k}{\bar{c}_p \mu}\right)^{0.4} \frac{\mu}{k}\right]$	$\frac{X}{X_{H_2}}$	Liquids	$\left[\frac{1}{\rho^2 \bar{c}_p^2}\left(\frac{k}{\bar{c}_p \mu}\right)^{0.4} \frac{\mu}{k}\right]$	$\frac{X}{X_{H_2}}$
H_2	662	1.0	H_2O	0.0004425	1.0
He	3,315	5.1	*Na	0.0008770	2.0
H_2O	5,040	7.7	*Hg	0.0033500	7.6
CO_2	6,560	10.1	*Bi	0.0053000	12.0
Air	8,150	12.5	Glycol	0.0091900	20.8
N_2	8,740	13.4	Freon-12	0.0095700	21.6

* A different correlation for heat transfer coefficient has been used.